The Effect of Poisson Data Weighting on PMT Array Diagnostics

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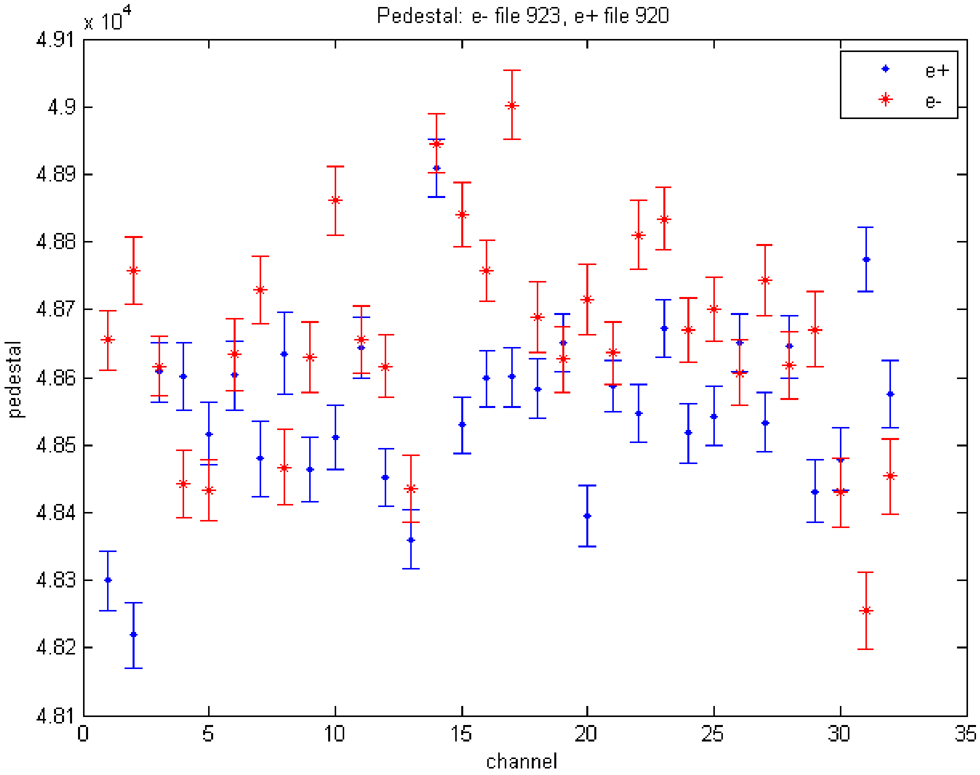
Pedestal Values Averaged over 9000 turns



Error bars represent the standard deviation of the pedestal data.
The above data is from CHESS in 9x6 operation.

The pedestal is subtracted from the measured signal for each channel and turn, giving $S = S_{background} + S_{actual}$. This is the signal to which a Gaussian is fitted.

Amplification Factor vs. Bunch

The signal can be expressed as

$$S = AN_\gamma,$$

where N_γ is the photons incident on a given channel and A is an undetermined amplification factor. According to Poisson statistics, the standard deviation is

$$\sigma = A\sqrt{N_\gamma}.$$

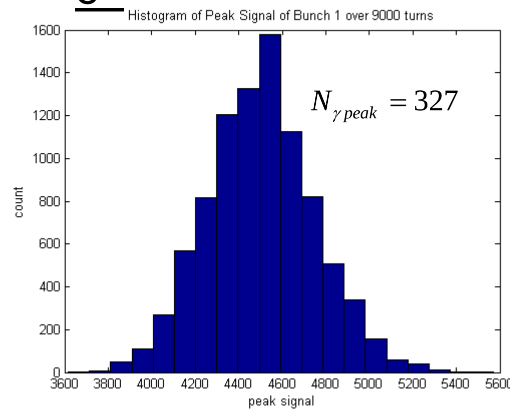
Since the peak signal for each bunch gives the most accurate estimate of A, it is used to determine the number of photons

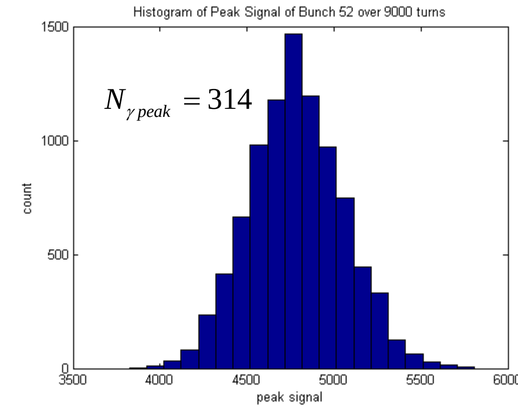
$$\mathrm{N}_\gamma = \left[\frac{S_p}{\sigma_p}\right]^2,$$

with which the amplification factor for each bunch is calculated:

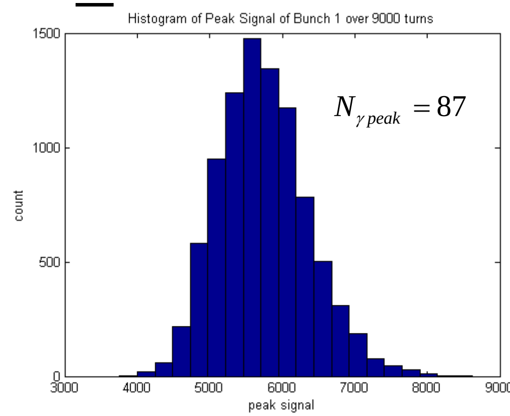
$$A = \frac{\sigma_p{}^2}{S_p}.$$

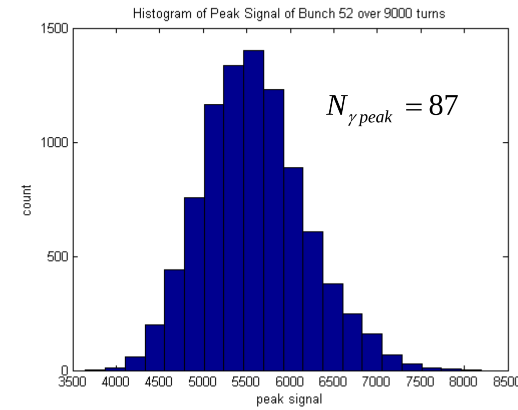
e+



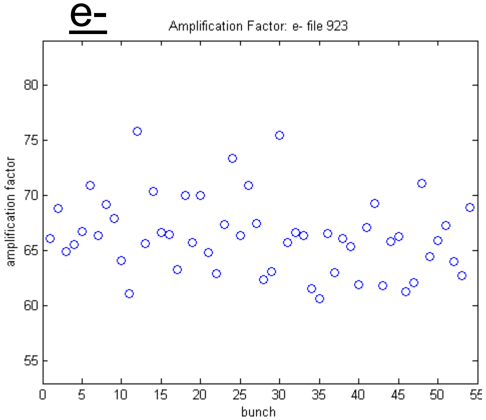


e-

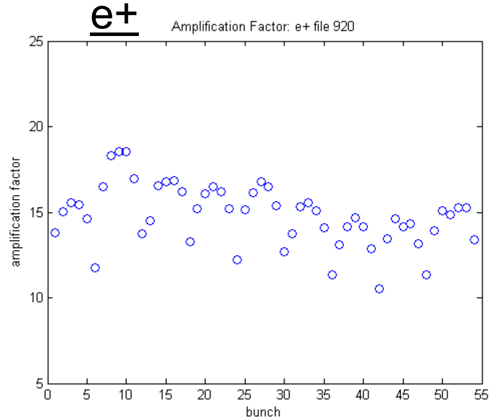




e-



e+

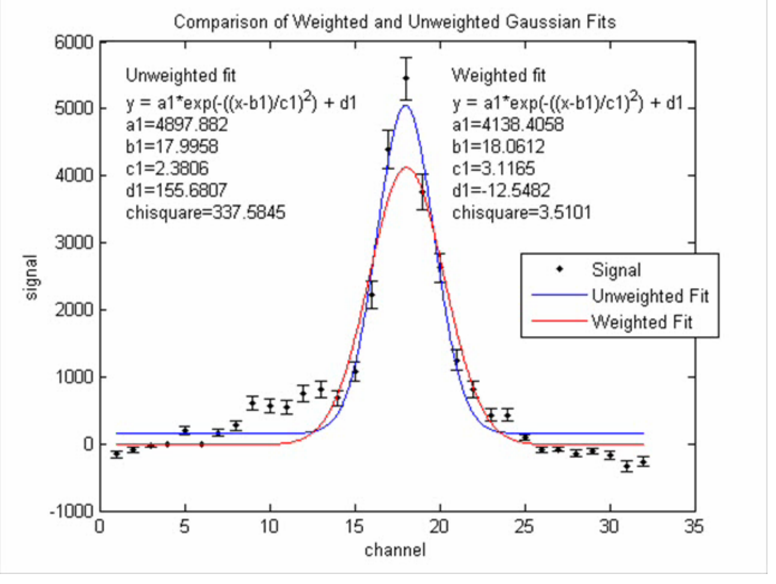


Comparison of Weighted and Unweighted Gaussian Fits: Flat Background

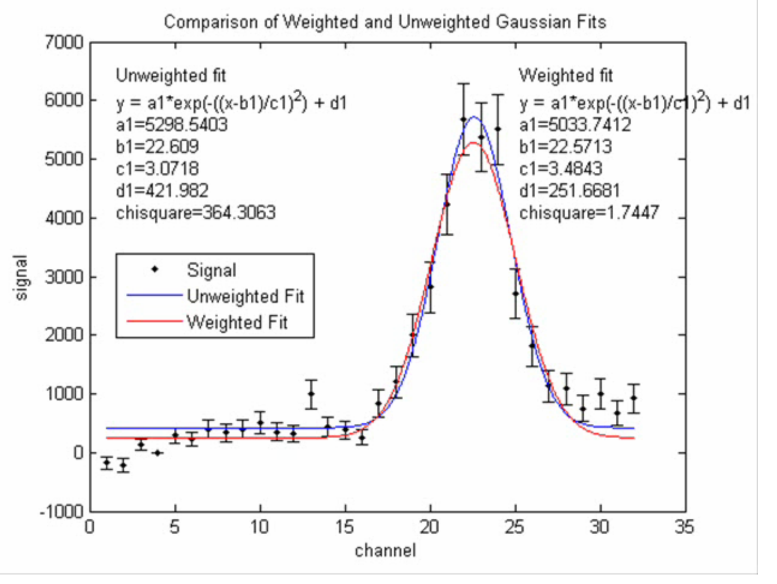
Each data point has error $\sigma = A\sqrt{N_\gamma} = \sqrt{A(S_{actual} + S_{background})}$.

The points are weighted by $\frac{1}{\sigma^2}$ and the Gaussian fit is determined.

e+



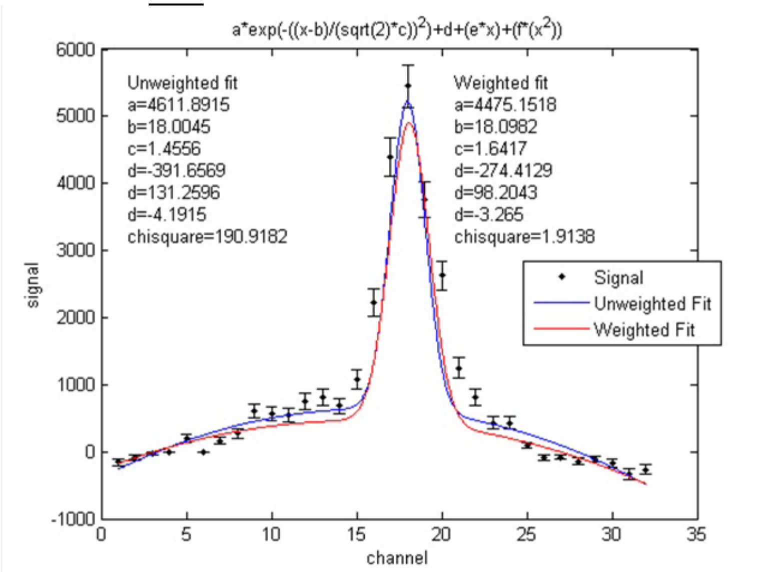
e-



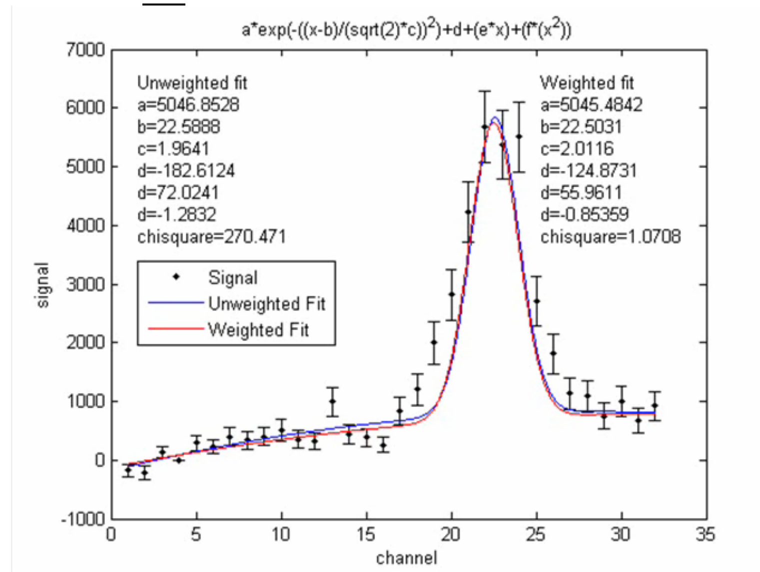
Sigma is larger when the data are weighted, especially for e+, due to reflection to the left of the peak.

Comparison of Weighted and Unweighted Gaussian Fits: Quadratic Background

e+



e-



With respect to the flat background with weighted fits, the quadratic background has smaller χ^2/ndf, indicating a fit that more closely matches PMT data.

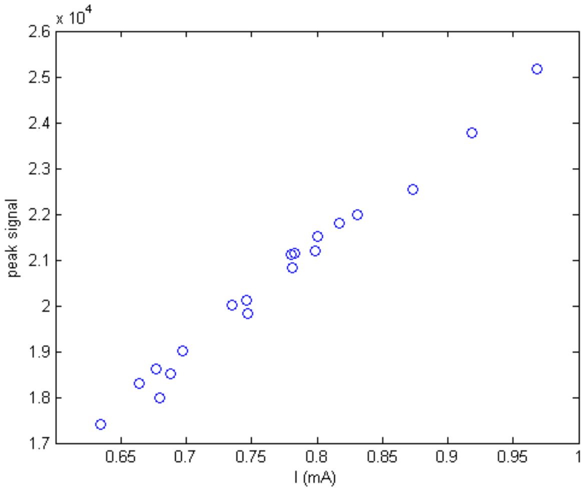
$$\chi^2 = \sum_{i=1}^{32} \frac{1}{\sigma_i^{\,2}} \left[S_i - f(x_i) \right]^2 \sim 1$$ suggests that error in fit is comparable to error intrinsic to data.

Relationship between Current and Average Peak Signal

e-

HV = -550 V

19 bunches

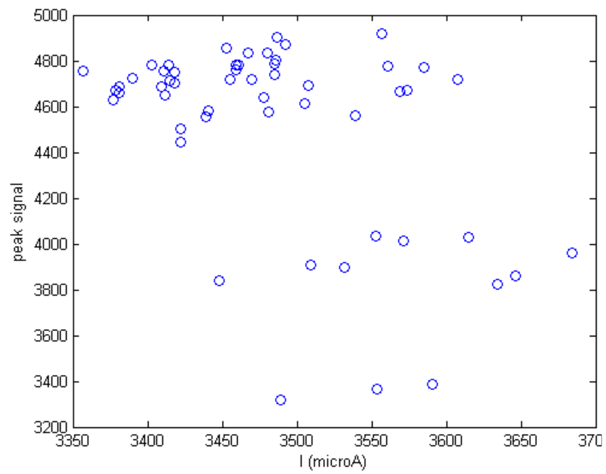


Strong linear correlation between current and peak signal.

e+

HV = -450 V

CHESS 9x6

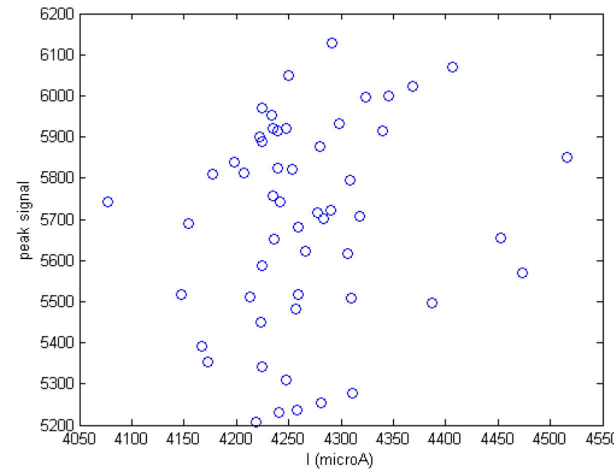


Not an obvious relationship between the two, perhaps due to bunch size increase that decreases the peak signal.

e-

HV = -450 V

CHESS 9x6

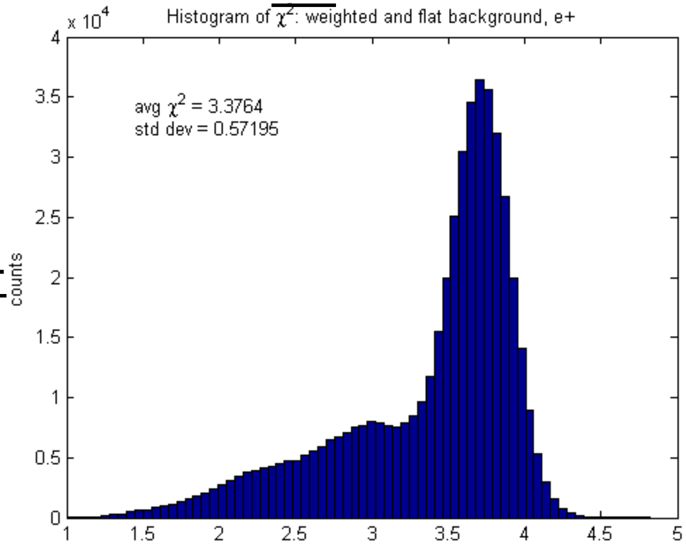


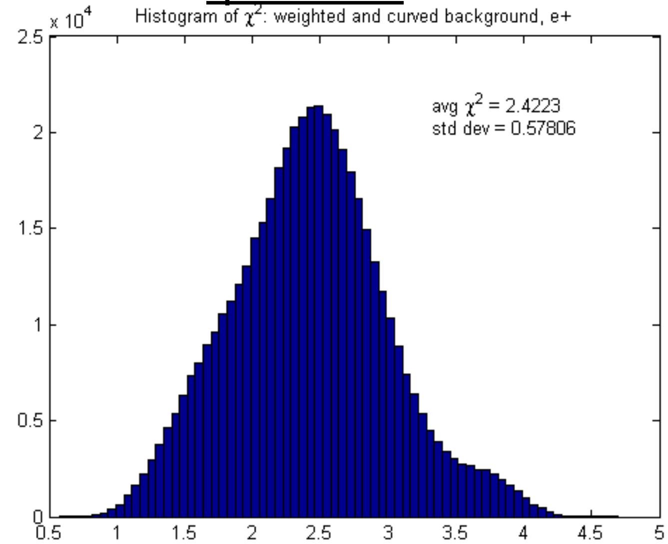
Not an obvious relationship between the two.

Comparison of χ^2/ndf: Flat vs. Quadratic Background

flat | quadratic

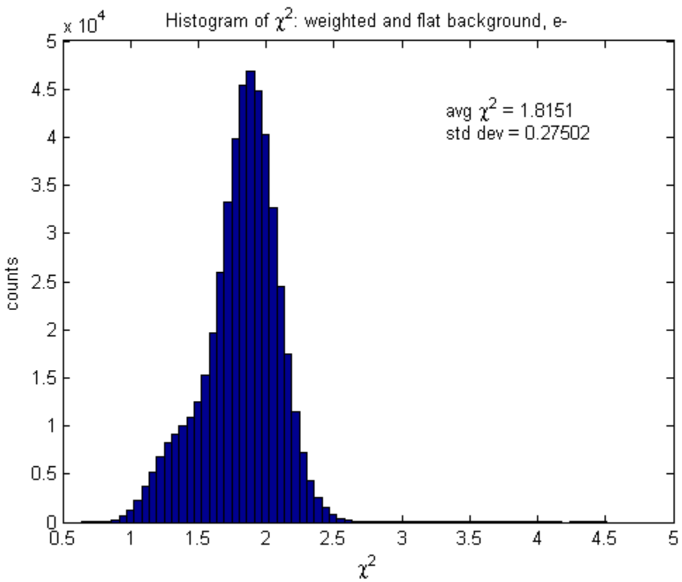
e+

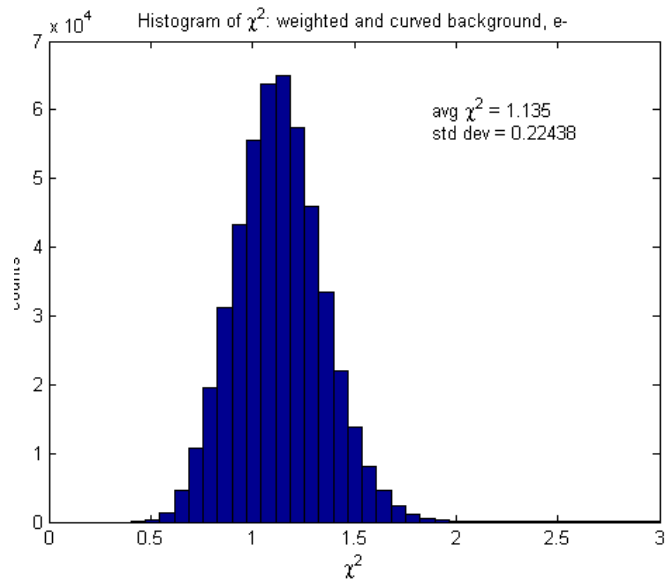




Quadratic background reduces χ^2/ndf by a factor of ~0.7.

e-



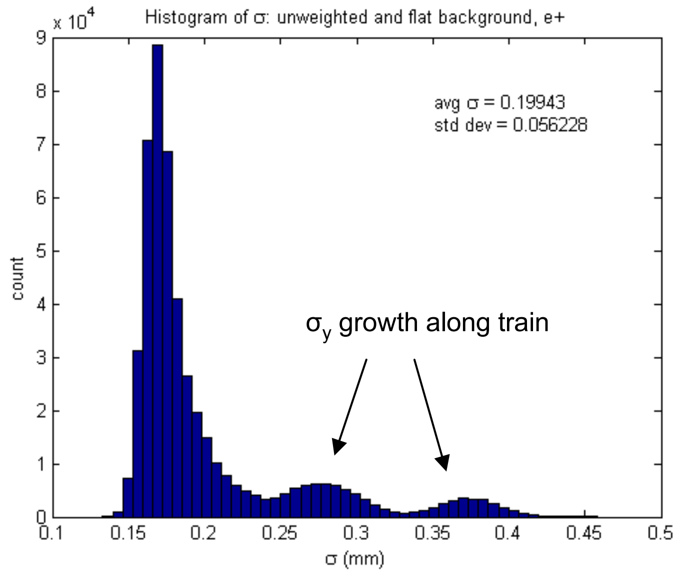


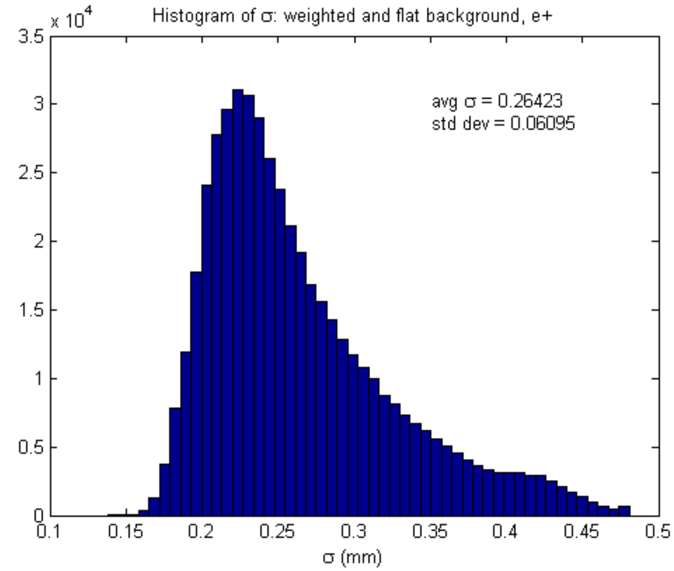
Quadratic background reduces χ^2/ndf by a factor of ~0.6.

Flat Background: How much does the measured σ_y change with weighted data?

unweighted | weighted

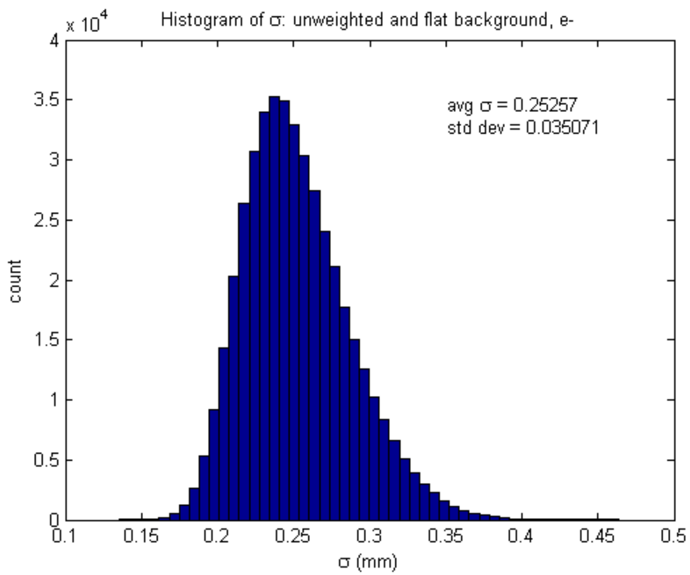
e+

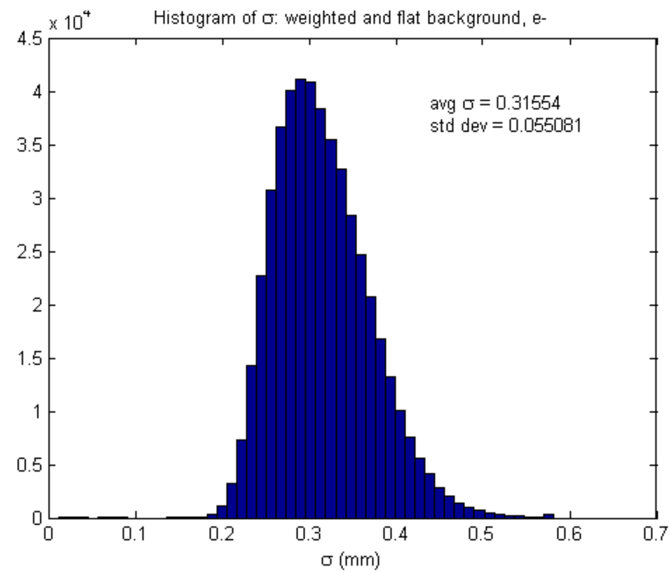




On average, σ_y increases by ~30% (flat background).

e-



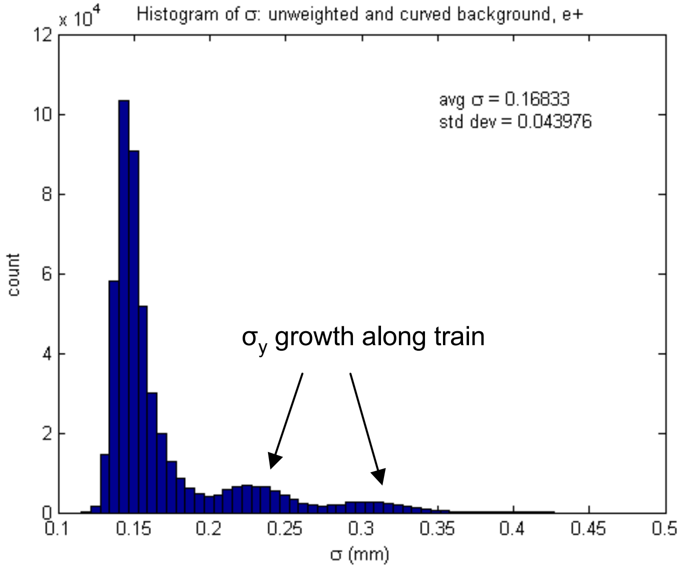


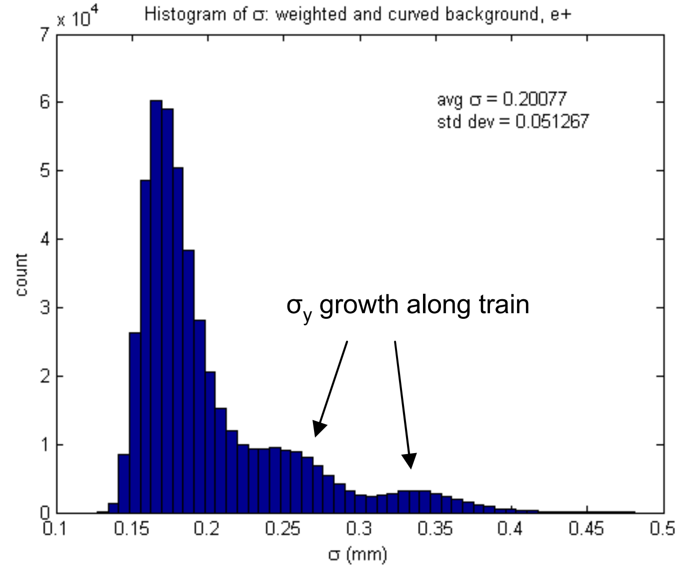
On average, σ_y increases by ~20% (flat background).

Quadratic Background: How much does the measured σ_y change with weighted data?

unweighted | **weighted**

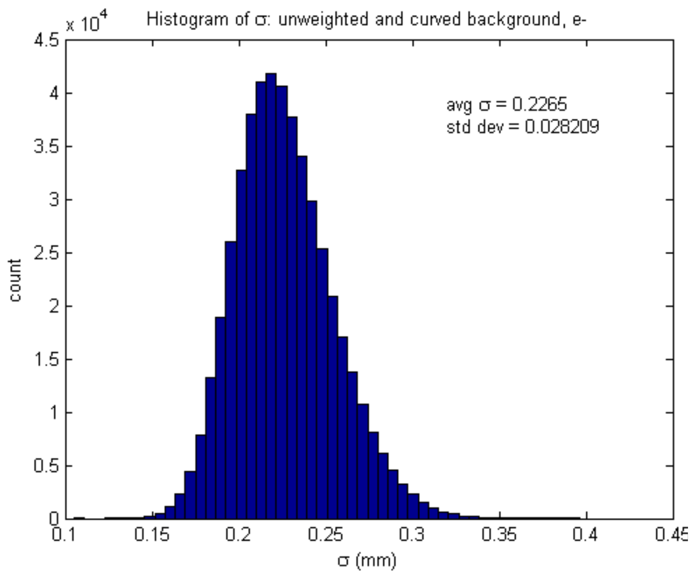
e+

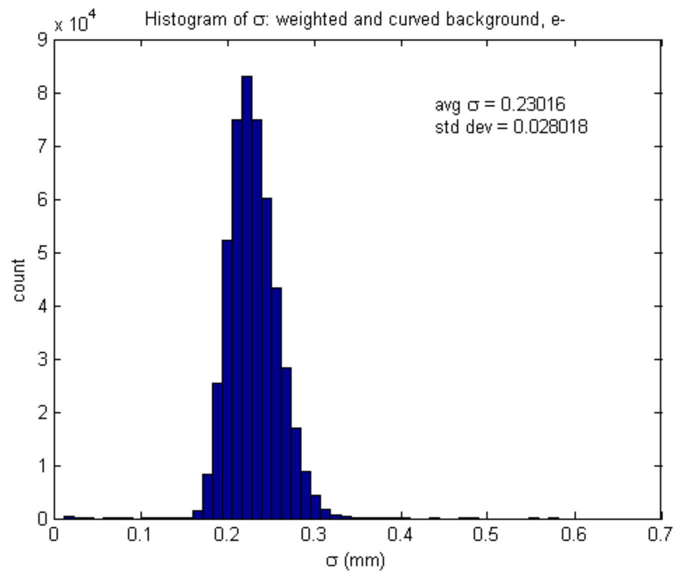




On average, σ_y increases by ~19%.

e-

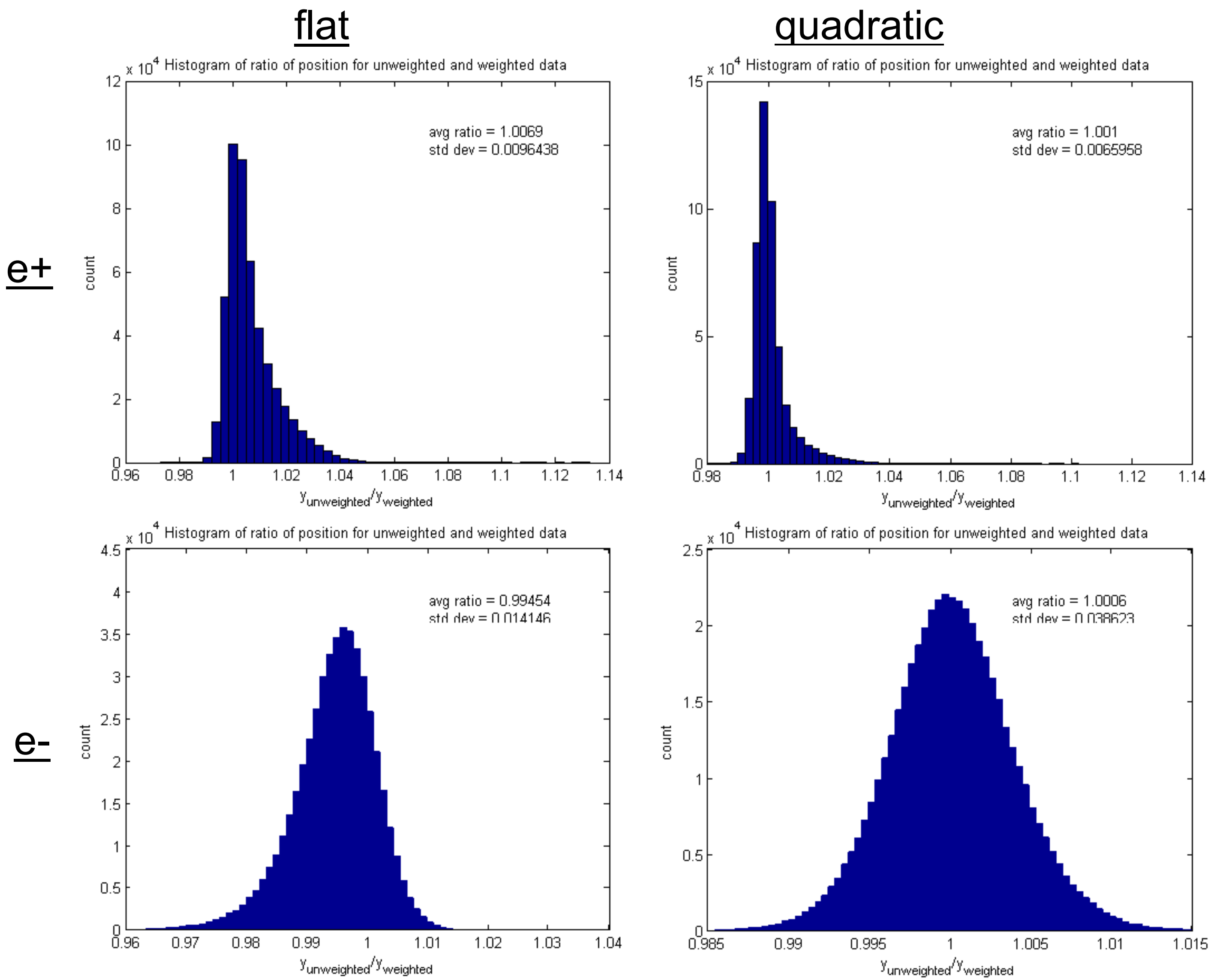




On average, σ_y increases by ~1.5%.

How much does the measured beam position change?

Let y be the mean of the Gaussian fit. The following are histograms of $r = y_{unweighted}/y_{weighted}$ for 54 bunches over 9000 turns.



The average ratio is ~1 for all data sets, implying that fitting to weighted data has little effect on the mean and hence the measured beam position.