# A Search for $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$ 

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#### Abstract

The branching fraction for a previously unobserved decay $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$is predicted theoretically in this document to be $0.65 \%$ of the branching fraction for the decay $D_{s}^{*+} \rightarrow D_{s}^{+} \gamma$. We conduct a search for the $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$in $586 \mathrm{pb}^{-1}$ of $e^{+} e^{-}$collision data collected with the CLEO-c detector at the Cornell Electron Storage Ring (CESR) operating at a center of mass energy of 4170 MeV and observe it with a significance of $6.39 \sigma$ over estimated backgrounds. The ratio of branching fractions $B\left(D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}\right) / B\left(D_{s}^{*+} \rightarrow D_{s}^{+} \gamma\right)$ is measured to be $0.72 \pm 0.14$ (stat) $\pm 0.05$ (syst) $\%$, which is within one standard deviation of uncertainty from the predicted value.


## 13 Systematic Uncertainties from the Tracking of Soft Electrons and Photons

As reported in Section [12] systematic errors in the measurement of $\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}$ will contribute to the systematic uncertainty in our measurement of the ratio of branching fractions $B\left(D_{s}^{*+} \rightarrow\right.$ $\left.D_{s}^{+} e^{+} e^{-}\right) B\left(D_{s}^{*+} \rightarrow D_{s}^{+} \gamma\right)$. In this section, we seek to estimate the systematic error in the measurement of $\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}$ in the energy range relevant for our analysis by studying the decay of $\psi(2 S)$ mesons to $J / \psi \pi^{0} \pi^{0}$. We estimate this systematic error by restricting the $e^{+} e^{-}$ energy to that found in $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$and measuring the ratio of the numbers of events where one of the $\pi^{0}$ Dalitz decays to $\gamma e^{+} e^{-}$to the number of events where both $\pi^{0}$ decay to $\gamma \gamma$ and comparing this to the ratio expected from currently accepted branching fractions for $\pi^{0} \rightarrow \gamma e^{+} e^{-}$and $\pi^{0} \rightarrow \gamma \gamma$.

Dataset 42 , which contains $53 \mathrm{pb}^{-1}$ of data taken at the $\psi(2 S)$ resonance, was used for this study. Since soft electrons from the Dalitz decay of the $\pi^{0}$ would also suffer from the systematic deviation in their energy and other track parameters if their tracks are pion-fitted, we reprocessed this dataset to include electron-fits. This has been described in Section 5

We tried to estimate $e^{+} e^{-}$reconstruction efficiency using the method of missing mass. This effort failed as the invariant mass of an electron is indistinguishable from that of a photon at our scale of energies and this makes it impossible for us to distinguish between efficient and inefficient events. In the following paragraphs, we describe a method that completely reconstructs the $\psi(2 S)$ from its decay into $J / \psi \pi^{0} \pi^{0}$ in order to estimate our systematic error in the measurement of $\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}$.

For our convenience, events where one of the $\pi^{0}$ Dalitz decays to $e^{+} e^{-} \gamma$ will be called events of Type I. Events where both $\pi^{0}$ decay to $\gamma \gamma$ will be called events of Type II. The latest fit in the Review of Particle Physics 2010 establishes the ratio $B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right) / B\left(\pi^{0} \rightarrow \gamma \gamma\right)$ to be $(1.188 \pm 0.034) \times 10^{-2}$. From this, we can establish that the ratio of numbers of these two types of events produced in our dataset should be ( $2.376 \pm 0.068$ ) \% from Eq. 76.

$$
\begin{equation*}
r=\frac{n_{I}}{n_{I I}}=2 \times \frac{B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)}{B\left(\pi^{0} \rightarrow \gamma \gamma\right.}=0.02376 \pm 0.00068 \tag{76}
\end{equation*}
$$

In our method, we obtain a measurement of this ratio from data and compute the branching fraction $B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)$. The deviation of this measurement from the currently accepted value of the branching fraction translates to the systematic uncertainty in our measurement of $\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}$ :

$$
\begin{equation*}
\frac{\Delta \epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}}{\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}}=\frac{\Delta B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)}{B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)} \tag{77}
\end{equation*}
$$

Our method reconstructs the $\psi(2 S)$ through events of Type I $\left(\psi(2 S) \rightarrow J / \psi \pi^{0} \pi^{0} ; \pi^{0} \rightarrow\right.$ $\left.\gamma \gamma ; \pi^{0} \rightarrow e^{+} e^{-} \gamma\right)$ and events of Type II $\left(\psi(2 S) \rightarrow J / \psi \pi^{0} \pi^{0} ; \pi^{0} \rightarrow \gamma \gamma ; \pi^{0} \rightarrow \gamma \gamma\right)$. We estimate the reconstruction efficiencies for both types of events using Monte Carlo samples. First, we establish a set of criteria to reconstruct Type I events in our data. To illustrate our method, we shall call the efficiency of selecting Type I events from an MC sample of Type I
events $\epsilon_{s}$. The efficiency of keeping Type II events in the signal region of these criteria from an MC sample of Type II events shall be called $\epsilon_{c}$. For $n_{I}$ produced Type I and $n_{I I}$ produced Type II events, we can expect an yield of $y$ events after applying this set of selection criteria to our data as expressed in Eq. 78,

$$
\begin{equation*}
n_{I} \epsilon_{s}+n_{I I} \epsilon_{c}=y \tag{78}
\end{equation*}
$$

Using the currently accepted ratio of $n_{I} / n_{I I}$ from Eq. 76, we may calculate $n_{I}$, the number of Type I events in our data, from this.

Hereafter, we construct a set of selection criteria to reconstruct Type II events in our data. Using Type II MC, we find out the reconstruction efficiency $\epsilon_{\gamma}$ for this set of criteria. Then we may estimate the number of produced Type II events in our data with this method as $n_{I I}$ using

$$
\begin{equation*}
n_{I I} \epsilon_{\gamma}=y_{\gamma} \tag{79}
\end{equation*}
$$

where $y_{\gamma}$ is the yield of our set of criteria on data to isolate Type II events.
Having estimated the number of Type I and II events in our data, we may estimate the branching fraction $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)$ using

$$
\begin{equation*}
B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=\frac{B\left(\pi^{0} \rightarrow \gamma \gamma\right)}{2} \frac{n_{I}}{n_{I I}} \tag{80}
\end{equation*}
$$

In order to establish a systematic uncertainty in our measurement of $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)$, we implement a second method for measuring this branching fraction. In this method, we use Type I and Type II events in our data that are most likely conversion events, events where one of the photons from the $\pi^{0}$ converts to a $e^{+} e^{-}$in material, in combination with Eq. 78 to estimate the total number of Type I and Type II events in the data. In order to select events that are most likely to be conversion events, we select events that are rejected by the $\delta d_{0}$ and $\delta \phi_{0}$ criteria on the tracks of the $e^{+} e^{-}$pair. These selection criteria have been described in Sections ?? and ??. The efficiency of selecting such conversion-type events from a MC sample of Type I events shall be called $\epsilon_{s}^{\prime}$. The efficiency of selecting such events from a MC sample of Type II events shall be called $\epsilon_{c}^{\prime}$. Thus, upon the application of our selection criteria (that inverts the standard $\delta d_{0}$ and $\delta \phi_{0}$ requirements), the yield in data may be denoted by $y^{\prime}$ as expressed in Eq. 81.

$$
\begin{equation*}
n_{I} \epsilon_{s}^{\prime}+n_{I I} \epsilon_{c}^{\prime}=y^{\prime} \tag{81}
\end{equation*}
$$

Solving Equations 78 and 79 simultaneously gives us the number of Type I and Type II events in the data. This ratio, $n_{I} / n_{I I}$, is plugged into Equation 80 to give us a second estimate for the $\pi^{0}$ Dalitz decay branching fraction.

Now we shall discuss the details of implementation of the two methods.


Figure 223: Invariant mass of the $J / \psi$ reconstructed from its decay to $e^{+} e^{-}$(top plots) and $\mu^{+} \mu^{-}$(bottom plots). The column on the left is from signal MC of Type I events. The column at the center is from signal MC of Type II events. The column on the right is from data.

### 13.1 Method 1

First, we shall describe the selection criteria used to select events from data in our first method.

The $J / \psi$ is reconstructed from its decays to $e^{+} e^{-}$and $\mu^{-} \mu^{+}$. The tracks of these leptons are fitted with the Kalman fitter using electron and muon mass hypotheses respectively. $50 \%$ of the expected number of hits on a track are required to be present. The momentum of each track is required to be between 500 MeV and 10 GeV . They may be reconstructed upto a $\cos \theta$ of 0.93 . The track parameter $d_{0}$ must be less than 5 mm and $z_{0}$ must be less than 5 cm . The $d E / d x$ of electron and muon tracks are required to be within $3 \sigma$ of their expected values. The $J / \psi$ has a mass of $3096.92 \pm 0.001 \mathrm{MeV}$ and a full natural width of $93.2 \pm 2.1$ keV . In our study, we require the invariant mass of the $e^{+} e^{-}$pair to be within 30 MeV of 3.09200 GeV , and the invariant mass of the $\mu^{-} \mu^{+}$pair to be within 30 MeV of 3.09692 GeV as depicted in Fig 223 ,

The first $\pi^{0}$ in Type I events is reconstructed from its decay to two photons. The photons must not have showered in known noisy crystals and must not have tracks matched to them. Each of their shower energies are required to be between 10 Mev and 2 Gev . The E9E25 unfolded $\left[^{*}\right]$ cut is required to be less than 1.0 . The pull mass of the $\pi^{0}$ is required to be


Figure 224: The invariant mass of the first $\pi^{0}$. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.




Figure 225: The invariant mass of the second $\pi^{0}$. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.
within $\pm 2.5 \sigma$. This is shown in Fig. 224.
The second $\pi^{0}$ in Type II events is reconstructed from its decay to a photon and a soft $e^{+} e^{-}$pair. Requirements on the photon are identical to those of the photons from the first $\pi^{0}$. The electron is Kalman fitted using the electron mass hypothesis and is required to have a momentum between 10 Mev and 2 GeV . It must be reconstructed within an angle of $\cos (\theta)=0.93$. The track parameter $d_{0}$ must be less than 5 mm and $z_{0}$ must be less than 5 cm . The $d E / d x$ of the track is required to be within $3 \sigma$ of the value expected of an electron. The invariant mass of the $\gamma e^{+} e^{-}$is required to be within 18 MeV of the nominal mass of the $\pi^{0}$ which is 134.9766 MeV . The distribution of this invariant mass and the selection range is shown in Fig. 225.

The electron and the positron are each required to have an energy less than 144 MeV as indicated in Fig. 226. This is the range of energies of the positron and the electron from the decay $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$.

Next, we combine the four-momenta of the $J / \psi$ and two $\pi^{0}$ to get the four-momentum


Figure 226: The distribution of energy of the positron and the electron from the Dalitz decay of the $\pi^{0}$ in the MC. Events containing positron and electrons with energy less than 144 MeV , as indicated, are accepted.
of the $\psi(2 S)$ meson. This must be close to the four-momentum of the colliding $e^{+} e^{-}$pair at the center of the CLEO-c detector. Hence, we apply selection criteria constraining each component of the momentum of the $\psi(2 S)$ to be within 40 MeV of that of the collision momentum. This is shown in Fig. 227,

We select events where the difference between the invariant masses of the reconstructed $\psi(2 S)$ candidate and the $J / \psi$ candidate is within 30 MeV of the nominal difference in masses. This is depicted in Fig. 228,

A background to the selection of Type I events are Type II events where one of the photons from a $\pi^{0}$ converts in material to produce an $e^{+} e^{-}$pair. We reject this background using the $\Delta d_{0}>-5 \mathrm{~mm}$ and $\Delta \phi_{0}<0.12$ criteria used in our $D_{s}^{*+} \rightarrow D_{s}^{+} e^{+} e^{-}$analysis. This is shown in Fig. 229 and 230

The aforementioned selection criteria are found to accept 1,069 Type I events out of a Monte Carlo sample of 299,794. Thus, we record the efficiency $\epsilon_{s}=0.0357 \pm 0.0011$ as applicable in Eq. 78, They are also found to accept 10 Type II events out of a Monte Carlo sample of 149,888 and thus we may write $\epsilon_{c}=2 / 149,888=(1.33 \pm 0.94) \times 10^{-5}$. When these selection criteria are applied to our data, we get an yield of $y=306$ events.

Assuming the established ratio of Type I to Type II events detailed in Eq. 76 to hold true, we may solve Eq. 78 for $n_{I}$. The solution is given in Eq. 82 and 83 . The $\oplus$ symbol is used to denote addition in quadrature. This gives us $n_{I}=8447 \pm 554$.

$$
\begin{gather*}
n_{I}=\frac{y}{\epsilon_{s}+\epsilon_{c} / r}  \tag{82}\\
\frac{\Delta n_{I}}{n_{I}}=\frac{\Delta y}{y} \oplus \frac{\Delta \epsilon_{s} \oplus\left(\epsilon_{c} / r\right)\left(\Delta \epsilon_{c} / \epsilon_{c} \oplus \Delta r / r\right)}{\epsilon_{s}+\epsilon_{c} / r} \tag{83}
\end{gather*}
$$

Having calculated the number of Type I events in our data, we may now estimate the number of Type II events present in the data sample. The reconstruction of Type II events is similar to the reconstruction of Type I events. The second $\pi^{0}$ is reconstructed from photons with the same selection criteria as the first $\pi^{0}$. The $\Delta d_{0}$ and $\Delta p h i_{0}$ cuts are not used as


Figure 227: Four momenta of $\psi(2 S)$ relative to the $e^{+} e^{-}$collision four momenta. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.


Figure 228: Difference between the invariant masses of the $\psi(2 S)$ and the $J / \psi$. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.


Figure 229: The $\Delta d_{0}$ between the $e^{+} e^{-}$pair from the second $\pi^{0}$. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.


Figure 230: The $\Delta \phi_{0}$ between the $e^{+} e^{-}$pair from the second $\pi^{0}$. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.
they are clearly inapplicable. A signal MC for Type II events was generated to calculate the signal efficiency of our criteria. Distributions of the $J / \psi$ mass, the pull masses of the two $\pi^{0}$, the momentum of the $\psi(2 S)$ relative to the collision momentum and the mass difference between the $\psi(2 S)$ and the $J / \psi$ are presented in Fig. 231, 232, 233, 234 and 235,

25,713 events out of 149,888 signal MC events were seen to be accepted by our criteria. This gives a signal efficiency $\epsilon_{I I}=0.1716+-0.0011$. We find the yield in data to be $y_{I I}=58,602$ events. Using Eq. 79 we infer that the number of Type II events is our data is $n_{I I}=341,607 \pm 2,555$.

Now, we may calculate the ratio of Type I to Type II events in our data as $n_{I} / n_{I I}$ and from that estimate the branching fraction $B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)$thus:

$$
\begin{equation*}
\frac{n_{I}}{n_{I I}}=\frac{8447 \pm 554}{341607 \pm 2555}=\frac{2 B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)}{(98.823 \pm 0.034) \times 10^{-2}} \tag{84}
\end{equation*}
$$

From this, we calculate $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=0.01222 \pm 0.00081$ (stat). In order to establish a systematic uncertainty in this measurement, we use a second method to estimate $B\left(\pi^{0} \rightarrow\right.$ $e^{+} e^{-} \gamma$ ).


Figure 231: Invariant mass of the $J / \psi$ reconstructed from its decay to $e^{+} e^{-}$(top plots) and $\mu^{+} \mu^{-}$(bottom plots). The column on the left is from signal MC of Type II events. The column on the right is from data.


Figure 232: The invariant mass of the first $\pi^{0}$. The column on the left is from signal MC of Type II events. The column on the right is from data.


Figure 233: The invariant mass of the second $\pi^{0}$. The column on the left is from signal MC of Type II events. The column on the right is from data.


Figure 234: Four momenta of $\psi(2 S)$ relative to the $e^{+} e^{-}$collision four momenta. The column on the left is from signal MC of Type II events. The column on the right is from data.


Figure 235: Difference between the invariant masses of the $\psi(2 S)$ and the $J / \psi$. The column on the left is from signal MC of Type II events. The column on the right is from data.


Figure 236: The $\Delta \phi_{0}$ between the $e^{+} e^{-}$pair. Now we accept events with $\Delta \phi_{0}$ greater than 0.12 . These were previously rejected as likely to be conversion-type events. The column on the left is from signal MC of Type I events. The column at the center is from MC of Type II events. The column on the right is from data.

### 13.2 Method 2

Our second method for estimating $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)$ uses conversion-type events found in data. Conversion-type events are those where both $\pi^{0}$ decay to $\gamma \gamma$ but at least one photon converts in material to form a $e^{+} e^{-}$pair. We select for such events by requiring all the criteria on $J / \psi$ and the invariant masses of the $\pi^{0}$ used to select Type I events, except now we look at the "wrong side" of the $\Delta d_{0}$ and $\Delta \phi_{0}$ criteria. In other words, we keep events which were previously being rejected by both the $\Delta d_{0}$ and the $\Delta \phi_{0}$ criteria. The distribution of $\Delta d_{0}$ is the same as Fig. [229 since all preceding criteria are identical. The distribution of $\Delta \phi_{0}$ after having accepted tracks on the "wrong side" of $\Delta d_{0}$ is presented in Fig. 236] respectively.

The efficiency of such a set of selection criteria for Type I events is found to be $\epsilon_{s}^{\prime}=$ $10 / 29,974=(3.34 \pm 1.1($ stat $)) \times 10^{-4}$. The efficiency for Type II events is found to be $\epsilon_{c}^{\prime}=54 / 149,888=(3.60 \pm 0.49($ stat $)) \times 10^{-4}$. On applying these selection criteria to our data, we are left with an yield of $y^{\prime}=141$ events. These values may be plugged into Eq. 81 and solved simultaneously with Eq. [78 to get $n_{I}=8437 \pm 342$. The solution for $n_{I}$ is given in Eq. 85 and 86

$$
\begin{gather*}
n_{I}=\frac{y \epsilon_{c}^{\prime}-y^{\prime} \epsilon_{c}}{\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}}  \tag{85}\\
\Delta n_{I}=\frac{\delta n_{I}}{\delta y} \Delta y \oplus \frac{\delta n_{I}}{\delta y^{\prime}} \Delta y^{\prime} \oplus \frac{\delta n_{I}}{\delta \epsilon_{c}} \Delta \epsilon_{c} \oplus \frac{\delta n_{I}}{\delta \epsilon_{s}} \Delta \epsilon_{s} \oplus \frac{\delta n_{I}}{\delta \epsilon_{c}^{\prime}} \Delta \epsilon_{c}^{\prime} \oplus \frac{\delta n_{I}}{\delta \epsilon_{s}^{\prime}} \Delta \epsilon_{s}^{\prime} \tag{86}
\end{gather*}
$$

where

$$
\frac{\delta n_{I}}{\delta y}=\epsilon_{c}^{\prime}
$$

$$
\begin{gathered}
\frac{\delta n_{I}}{\delta y^{\prime}}=-\epsilon_{c} \\
\frac{\delta n_{I}}{\delta \epsilon_{c}}=\frac{-y}{\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}}+\frac{y \epsilon_{c}^{\prime}-y^{\prime} \epsilon_{c}}{\left(\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}\right)^{2}} \epsilon_{s}^{\prime} \\
\frac{\delta n_{I}}{\delta \epsilon_{s}}=-\frac{y \epsilon_{c}^{\prime}-y^{\prime} \epsilon_{c}}{\left(\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}\right)^{2}} \epsilon_{c}^{\prime} \\
\frac{\delta n_{I}}{\delta \epsilon_{c}^{\prime}}=\frac{y}{\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}}-\frac{y \epsilon_{c}^{\prime}-y^{\prime} \epsilon_{c}}{\left(\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}\right)^{2}} \epsilon_{s} \\
\frac{\delta n_{I}}{\delta \epsilon_{s}^{\prime}}=\frac{y \epsilon_{c}^{\prime}-y^{\prime} \epsilon_{c}}{\left(\epsilon_{s} \epsilon_{c}^{\prime}-\epsilon_{c} \epsilon_{s}^{\prime}\right)^{2}} \epsilon_{c}
\end{gathered}
$$

Now, we may calculate the ratio of Type I to Type II events in our data as $n_{I} / n_{I I}$ and from that estimate the branching fraction $B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)$thus:

$$
\begin{equation*}
\frac{n_{I}}{n_{I I}}=\frac{8437 \pm 342}{341607 \pm 2555}=\frac{2 B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)}{(98.823 \pm 0.034) \times 10^{-2}} \tag{87}
\end{equation*}
$$

From this, we calculate $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=0.01220 \pm 0.00050$ (stat).
Now, we may combine our results from the two methods to establish a systematic error. Result from method 1: $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=0.01222 \pm 0.00081$ (stat). Result from method 2: $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=0.01220 \pm 0.00050$ (stat). The result of method 2 has the smaller uncertainty and will, therefore, be quoted as the central value of our measurement. The statistical uncertainty will quoted as the quadrature sum of the uncertainties in the two results. The absolute difference between the central values of the two results will be quoted as the systematic uncertainty in our measurement. Hence, we report $B\left(\pi^{0} \rightarrow e^{+} e^{-} \gamma\right)=$ $(1.222 \pm 0.081$ (stat) $\pm 0.002$ (syst) $) \times 10^{-2}$.

As quoted previously, the currently accepted branching fraction for the Dalitz decay of the $\pi^{0}$ is $(1.174 \pm 0.035) \times 10^{-2}$. The difference between this and our result is $0.046 \%$. Hence, we cannot motivate a correction to the tracking efficiency and must settle for an uncertainty. We add the difference between our measured branching fraction and the currently accepted measurement, and the uncertainties in our result in quadrature to get a total uncertainty of $0.077 \%$. Thus, the fractional uncertainty that we set out to estimate is found to be $6.51 \%$ as shown in Eq. 88

$$
\begin{equation*}
\frac{\Delta \epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}}{\epsilon_{e^{+} e^{-}} / \epsilon_{\gamma}}=\frac{\Delta B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)}{B\left(\pi^{0} \rightarrow \gamma e^{+} e^{-}\right)}=\frac{0.077 \%}{1.174 \%}=6.51 \% \tag{88}
\end{equation*}
$$

## References

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