CBPM raw button distribution fit: convolution approach

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CESR acc group meeting

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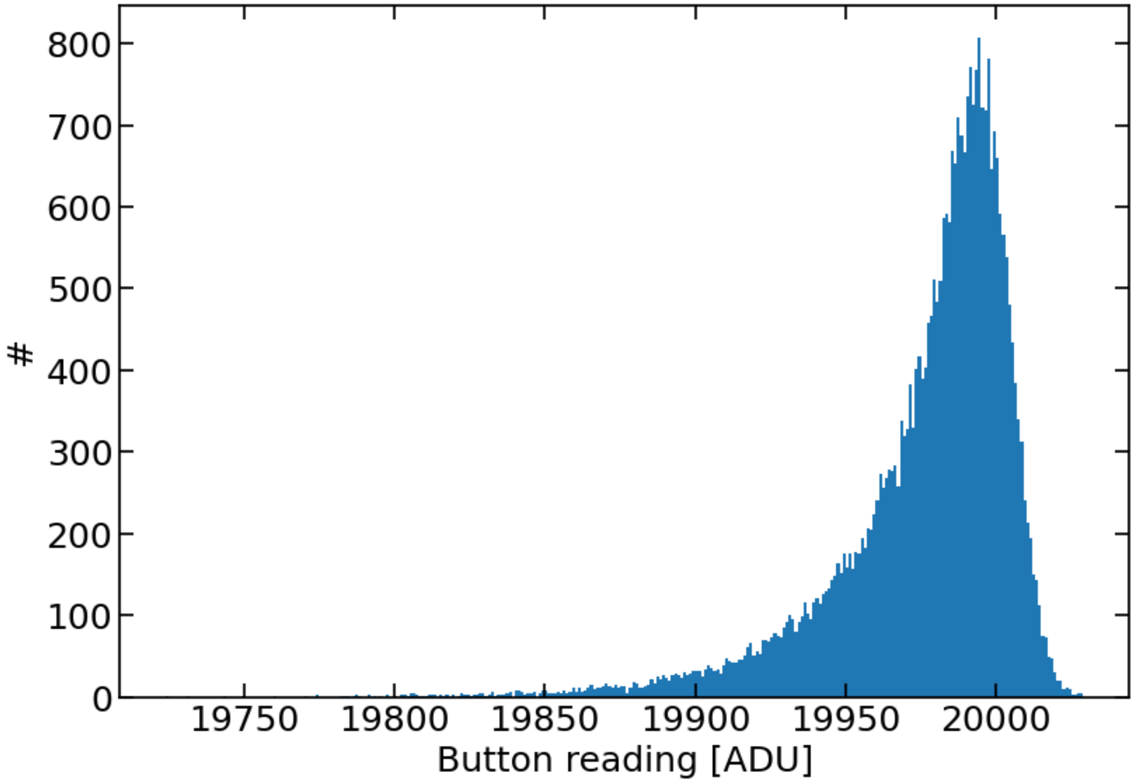
CBPM raw button distribution

CBPM raw button distribution looks typically like this simulated one:



The shape is determined by the the three known sources of errors:

- ✗ off-peak sampling
- ✗ clock jitter
- ✗ electronics noise

Fitting button distribution

Successfully fitting the distribution could allow things such as:

- ✗ button data quality
- ✗ in situ uncertainty measurement
- ✗ beam centroid modulation measurement

The button distribution's shape is the result of:

$$A \cdot \alpha_{gain} \cdot \boxed{\alpha_{noise}} \circledast cos\left[\omega(\boxed{t_j} + t_o)\right]$$

Gaussian distribution

Gaussian distribution

Sum of convoluted PDF

From **Suntao**:

$$A \cdot \alpha_{gain} \cdot \alpha_{noise} \cdot cos\left[\omega(t_j + t_o)\right]$$

Probability density function (PDF) is a Gaussian function f(x)

$$f(x) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}$$

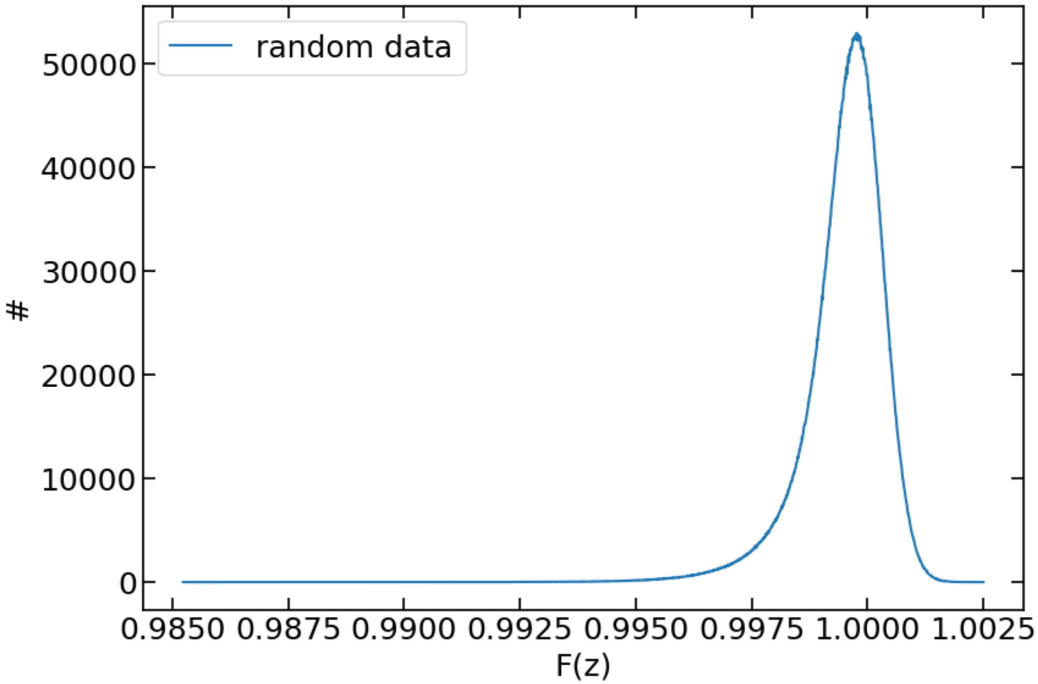
Probability density function (PDF) is function g(x)

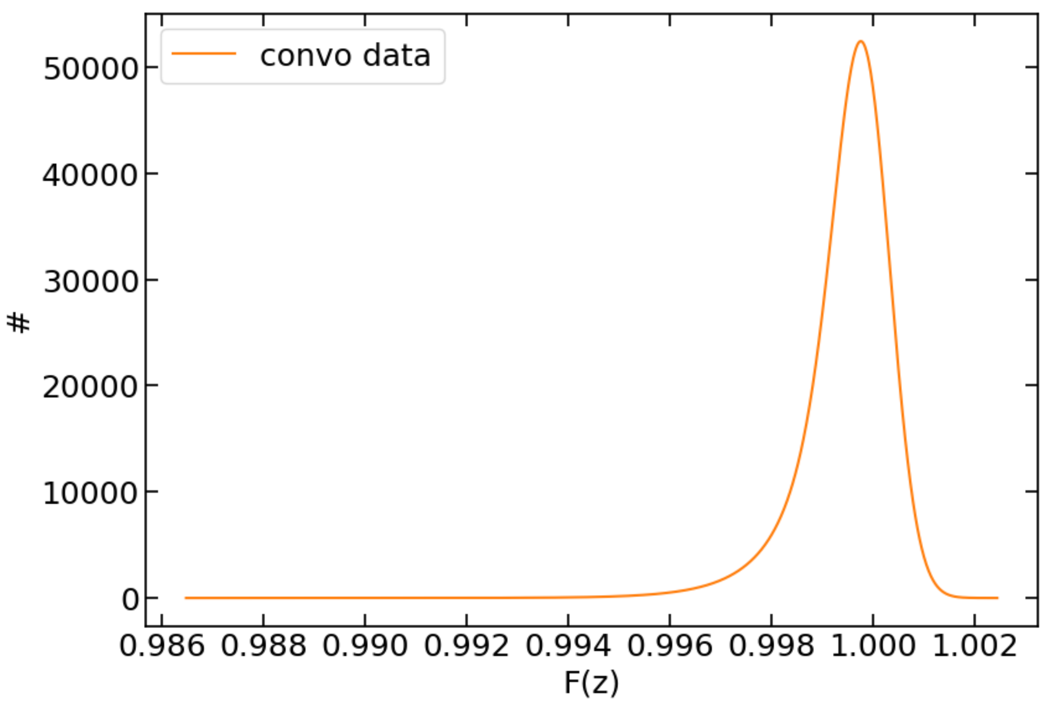
$$g(x) = \frac{e^{-\frac{(\mathrm{acos}(x)-\omega t_0)^2}{2(\omega\sigma)^2}}}{\omega\sigma\sqrt{2\pi(1-x^2)}}$$ x within [-1,1]

So the PDF of total equation (product of two functions) is $F(z) = \int_{-1}^{1} g(x) f\left(\frac{Z}{x}\right) dx/x$ (Mellin convolution)

The analytical form of F(z) is complicated but it can be solved numerically.
So it may be defined as a function or subroutine and used for fitting.

My implementation of **Suntao**'s work





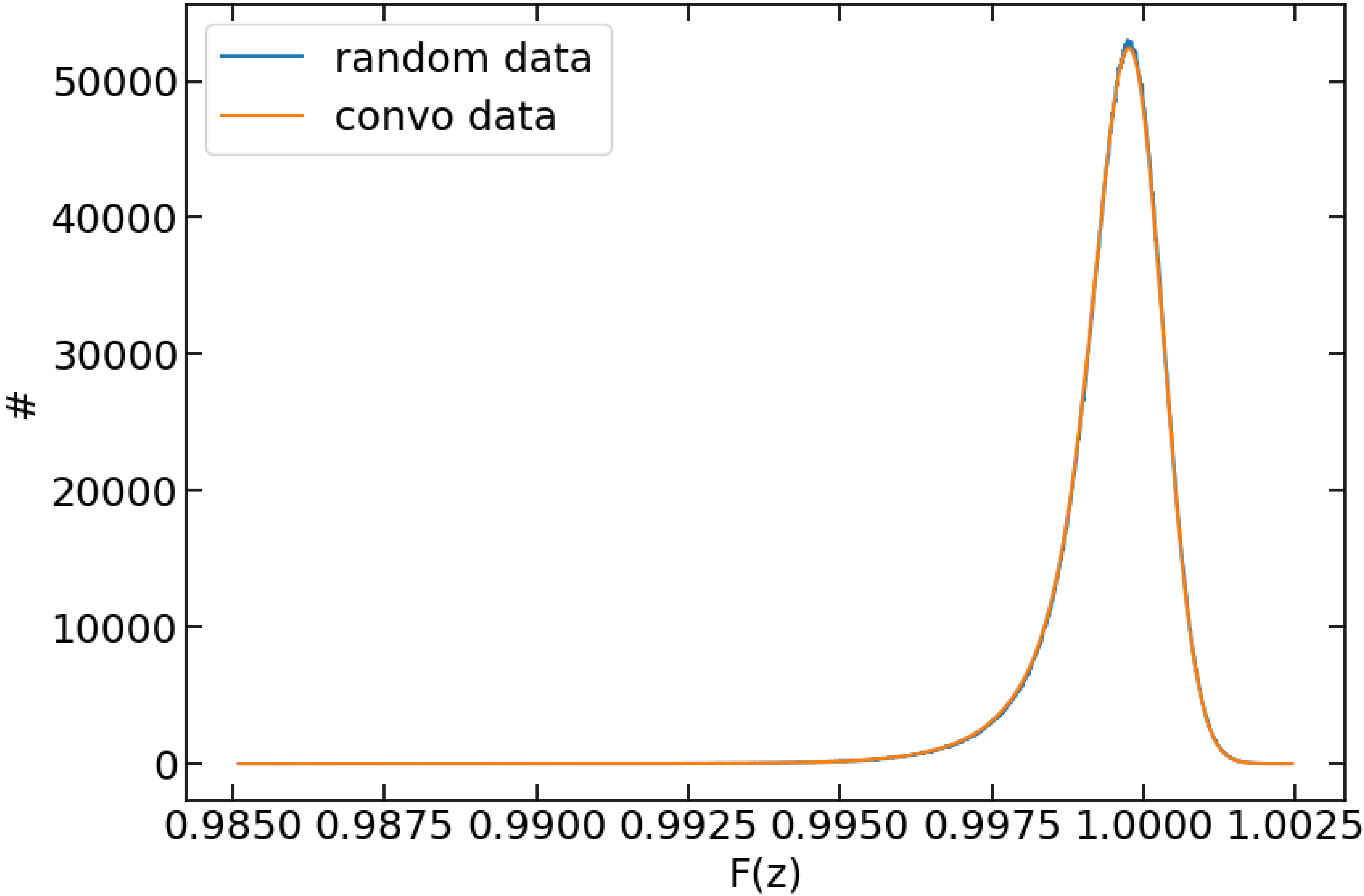
<u>Parameters:</u>

jitter = 10 ps
offset = 0 ps
omega = 2π.500 MHz
snr = 10/20,000 = 5e-4

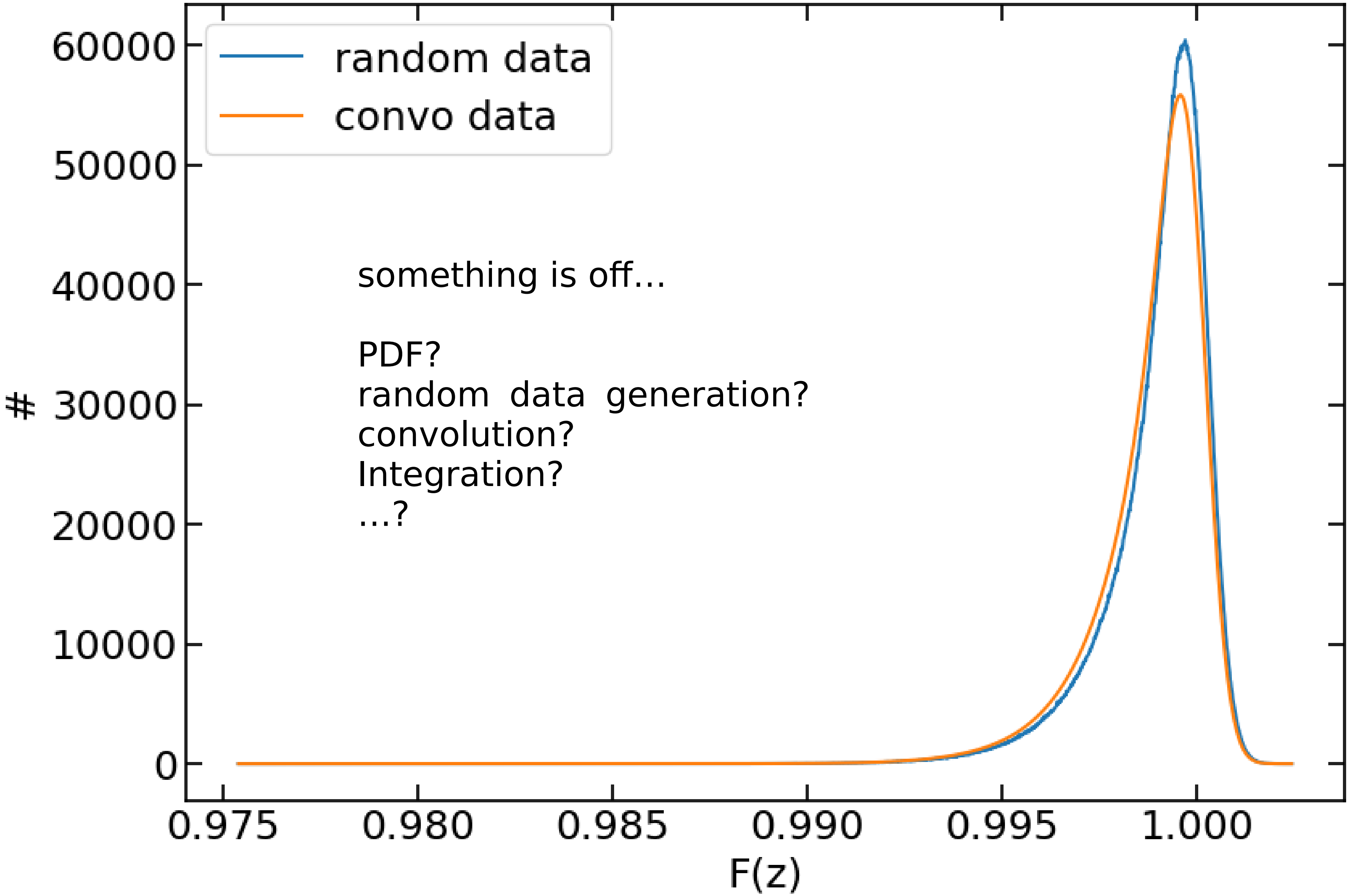
<u>Convolution setting:</u>

x integrated over [0.9, 1]
x step = 1e-5

My implementation of **Suntao**'s work



Changing offset **from 0 to 10 ps**



Additional materials