In situ fitting of instrumental uncertainties

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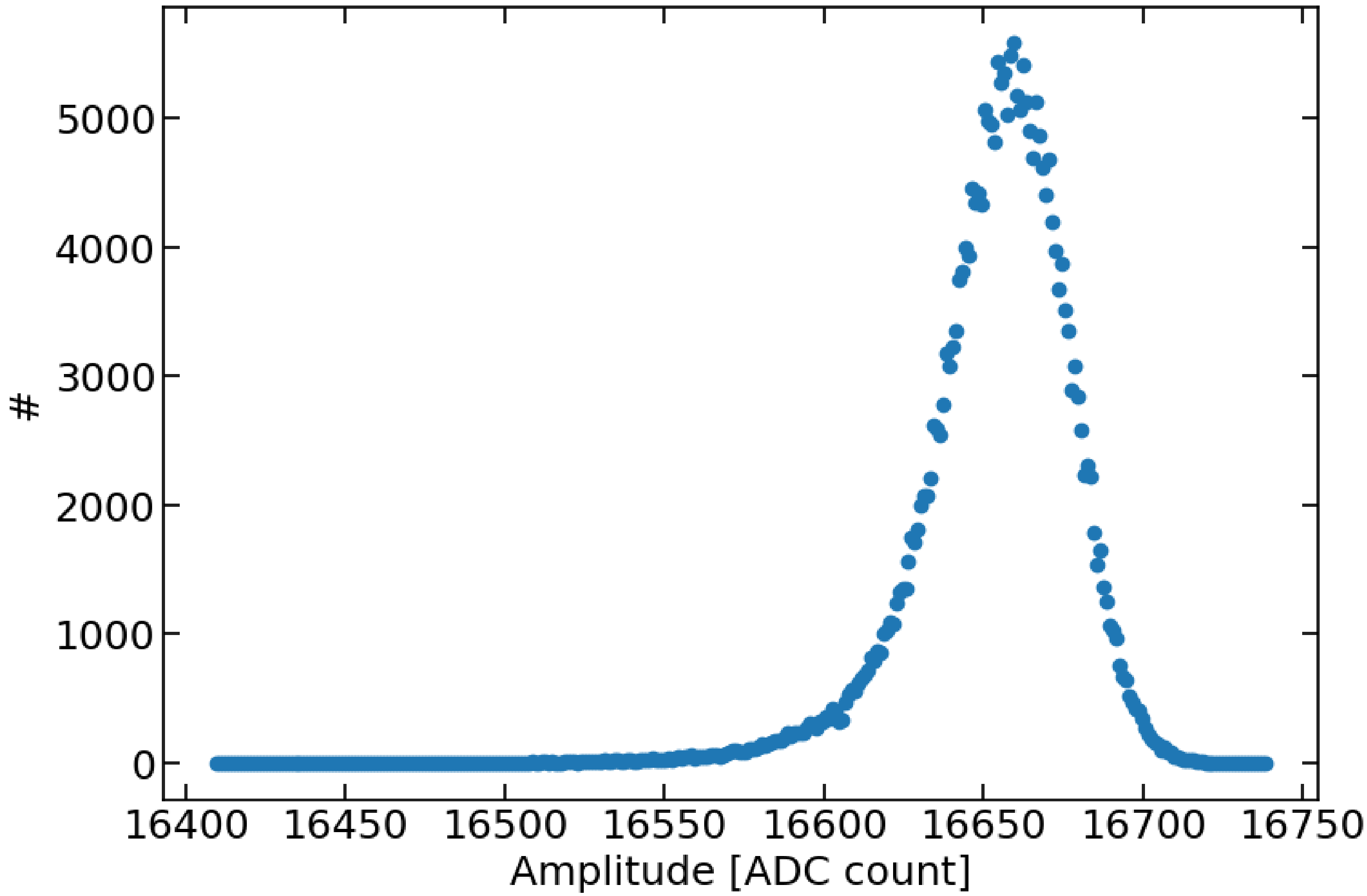
CESR acc. group meeting: April 26, 2023

Signal and its uncertainty

Measure button signal amplitude for many, many turns and build distribution



Assumed so far that the per-bin uncertainty followed Poisson: $\sqrt{n}$

In probability theory and statistics, the **Poisson distribution** is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.[1] It is named after French mathematician Siméon Denis Poisson (/ˈpwɑːsɒn/;

In probability theory and statistics, the **binomial distribution** with parameters *n* and *p* is the discrete probability distribution of the number of successes in a sequence of *n* independent experiments, each asking a yes–no question, and each with its own Boolean-valued outcome: *success* (with probability *p*) or *failure* (with probability $q = 1 - p$). A single success/failure experiment is also

Key differences:

Poisson statistics is for "**self-occuring**" event, Binomial is for triggered **events**

Poisson variance = **n**, Binomial variance = **npq**

Multinomial

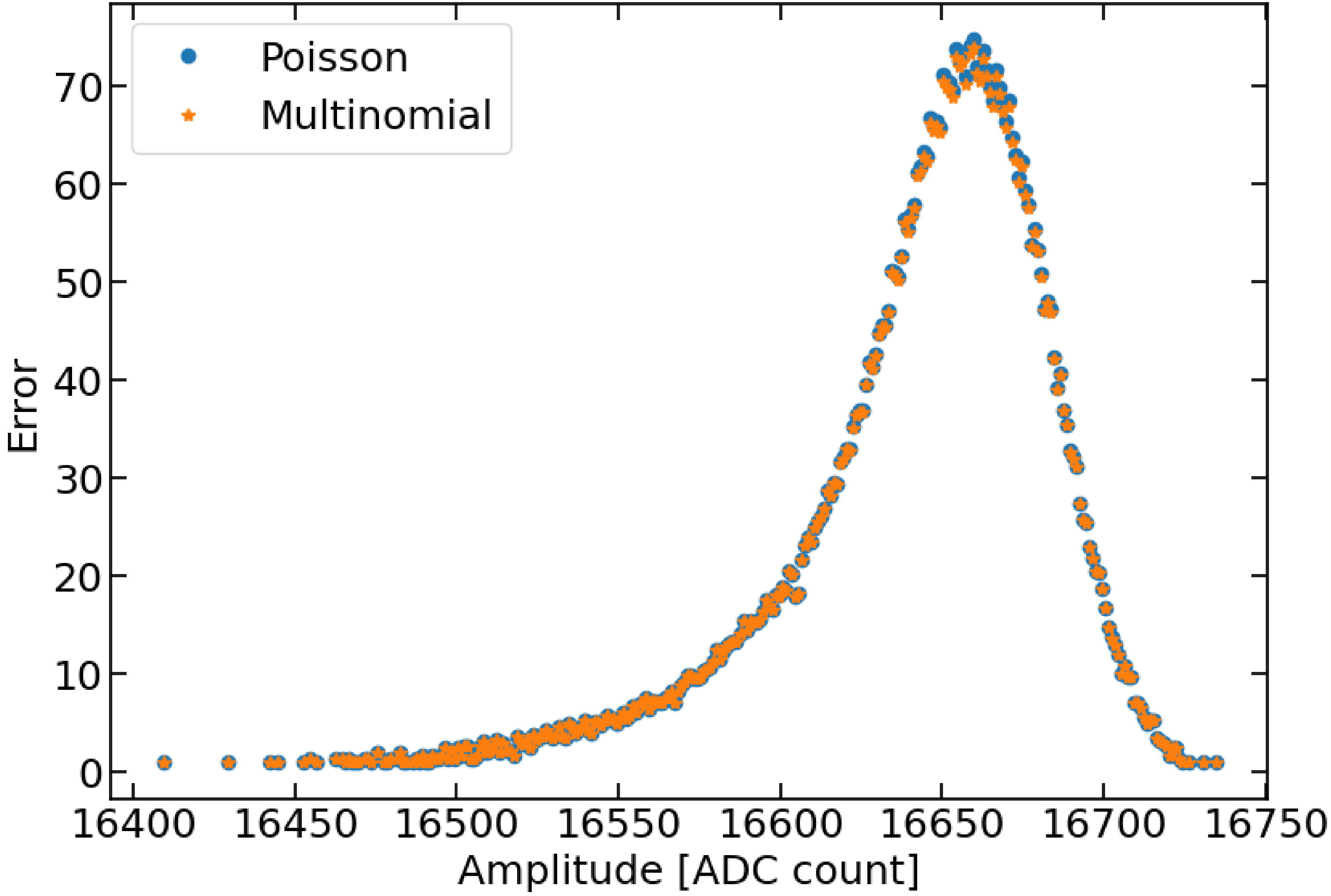
Each bin in our distribution represent a Binomial statistics, thus the entire distribution follow a multinomial statistics

Let k be a fixed finite number. Mathematically, we have k possible mutually exclusive outcomes, with corresponding probabilities $p_1, \ldots, p_k$, and n independent trials. Since the k outcomes are mutually exclusive and one must occur we have $p_i \geq 0$ for $i = 1, \ldots, k$ and $\sum_{i=1}^{k} p_i = 1$. Then if the random variables X_i indicate the number of times outcome number i is observed over the n trials, the vector $X = (X_1, \ldots, X_k)$ follows a multinomial distribution with parameters n and $\mathbf{p}$, where $\mathbf{p} = (p_1, \ldots, p_k)$. While the trials are independent, their outcomes X_i are dependent because they must be summed to n.

In simple terms:

- ✗ the probability p_i in a given bin i is its number of entries divided by the total number of entries in the distribution
- ✗ Variance = $\mathbf{n p_i q}$

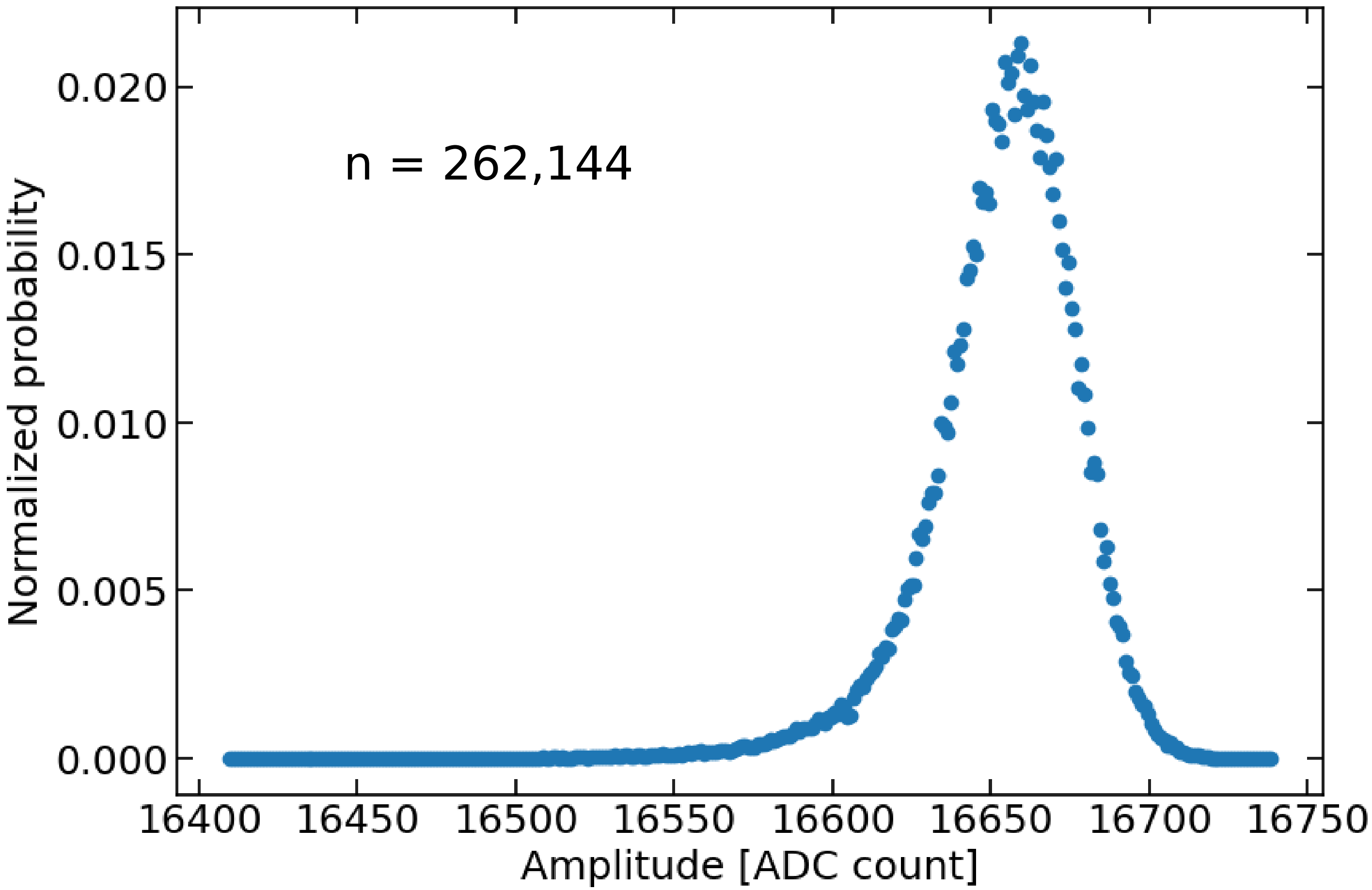
Poisson vs Multinomial



Turns out that yes Poisson is approximating well the Multinomial statistics

The Poisson distribution can be derived as a limiting case to the binomial distribution as the number of trials goes to infinity and the expected number of successes remains fixed — see law of rare events below. Therefore, it can be used as an approximation of the binomial distribution if n is sufficiently large and p is sufficiently small. The Poisson distribution is a good approximation of the binomial distribution if n is at least 20 and p is smaller than or equal to 0.05, and an excellent approximation if $n \geq 100$ and $n\,p \leq 10$.[28]

$$F_{\mathrm{Binomial}}(k;n,p) \approx F_{\mathrm{Poisson}}(k;\lambda=np)$$



Additional materials