Emittance Reach for Operating CESR as a Damping Ring Test Facility

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October 23, 2005

The minimum horizontal emittance that can be achieved for a suitable wiggler configuration in the Cornell Electron Storage Ring will determine its suitability for operation as a damping ring test facility, CesrTF. This memo presents an analytical estimate of the emittance and compares it with detailed numerical results for an optimized CesrTF lattice [1].

1 Emittance Limit in a Wiggler-Dominated Ring

The expression for the equilibrium horizontal emittance in an electron storage ring is given by [2]:

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x} \frac{\mathcal{I}_{5x}}{\mathcal{I}_2}, \tag{1}$$

where $C_q = 3.8319 \times 10^{-13}$m, γ is the usual relativistic factor, J_x is the horizontal damping partition number, and $\mathcal{I}_2$ and $\mathcal{I}_{5x}$ are synchrotron radiation integrals. The radiation integrals, for a ring with only horizontal bends, are defined as [3]:

$$I_2 \left[m^{-1}\right] = \oint \rho^{-2} ds \tag{2}$$

and

$$\mathcal{I}_5 \left[m^{-1}\right] = \oint \frac{\mathcal{H}_x}{|\rho|^3} ds \tag{3}$$

where the integrals are over the circumference of the ring and the expression for "curly-H" is $\mathcal{H}_x = \beta_x^{-1} \left[\eta_x^2 + (\beta_x \eta_x' + \alpha_x \eta_x)^2\right]$. η_x and η_x' are the horizontal dispersion and its derivative and α_x and β_x are the horizontal Twiss parameters. Each integral can be broken into pieces over the normal bending region of the ring and the wiggler region of the ring. In the limit of a wiggler-dominated ring, we expect the wiggler pieces of each integral to dominate and thus we will consider only the wiggler portion of the integrals in the following estimate.

Given the form of $\mathcal{H}$, it is clear that the wiggler contribution can be minimized by placing the wigglers in zero dispersion insertion regions. However, due to the bending generated by the wigglers themselves, the dispersion cannot be truly zero and hence there is a minimum achievable emittance that can be obtained from the wigglers alone. At this point we closely parallel the derivation of Wiedemann [2] to estimate this wiggler-only emittance limit.

Consider a sinusoidal wiggler whose field is given by:

$$B(s) = B_w \cos k_p s, \tag{4}$$

where B_w is the peak wiggler field and $k_p = 2\pi/\lambda_p$. λ_p is the wiggler period. Using $\rho^{-1}(s) = \rho_w^{-1}\cos k_p s$, the differential equation for the dispersion within the wiggler is then:

$$\eta_x'' = \frac{1}{\rho_w}\cos k_p s, \tag{5}$$

where ρ_w is the radius of curvature at peak wiggler field. Solving for η_x gives:

$$\eta_x(s) = \frac{1}{k_p^2\rho_w}\left(1-\cos k_p s\right). \tag{6}$$

At this point let's assume that β_x is approximately constant within each wiggler. In this limit the α_x term of $\mathcal{H}$ can be neglected. We can now write the contribution to $\mathcal{I}_5$ from a half-period of the wiggler as:

$$\Delta\mathcal{I}_5 = \int_0^{\lambda_p/2}\frac{|\cos^3 k_p s|}{\rho_w^3}\left[\frac{(1-\cos k_p s)^2}{\beta_x k_p^4\rho_w^2} + \frac{\beta_x\sin^2 k_p s}{k_p^2\rho_w^2}\right]ds \tag{7}$$

which, upon integration, yields:

$$\Delta\mathcal{I}_5 = \frac{36}{15\beta_x k_p^5\rho_w^5} + \frac{4\beta_x}{15k_p^3\rho_w^5} \approx \frac{4\beta_x}{15k_p^3\rho_w^5} \tag{8}$$

where we have assumed $\beta_x \gg \lambda_p$. The contribution to $\mathcal{I}_2$ for a wiggler half-period can be similarly evaluated,

$$\Delta\mathcal{I}_2 = \frac{\pi}{2k_p\rho_w^2}, \tag{9}$$

and we immediately obtain the expression for the minimum emittance [5, 6] in a wiggler-dominated storage ring:

$$\varepsilon_x \approx C_q\frac{\gamma^2}{J_x}\frac{8\beta_x}{15\pi k_p^2\rho_w^3} \tag{10}$$

Nominal CESR wiggler parameters are $B_w = 1.9$ T and $\lambda_p = 0.4$ m. Setting $J_x = 1$ with $E_{beam} = 2.0$ GeV and $\beta_x = 15$ m gives $\varepsilon_x \approx 1.4$ nm.

2 Emittance Limit Estimate for CesrTF

The proposed CesrTF lattice design with 12 standard CESR 8-pole wigglers, 48 periods in total, located in low dispersion regions is the starting point for our damping ring test facility proposal. The dispersion and beta functions are shown in Figure 1. The wigglers are located in 3 low dispersion regions. There are 2 triplets in the arcs, at 129–133 m and 637.5–641.5 m, and 6 wigglers in the L3 insertion region located at 380.8–389.8 m. The average value of β_x for the 12 wigglers in this design is 13 m.

In order to verify the validity of the approximations above which led to Eqn. 10, we calculate the wiggler contributions to $\mathcal{I}_2$, $\mathcal{I}_5$ and the wiggler-dominated value of ε_x. These are

$$\begin{aligned}\mathcal{I}_2 &= 0.78\ \text{m}^{-1}\\ \mathcal{I}_5 &= 1.61\times10^{-4}\ \text{m}^{-1}\\ \varepsilon_x &= 1.22\ \text{nm}\end{aligned} \tag{11}$$

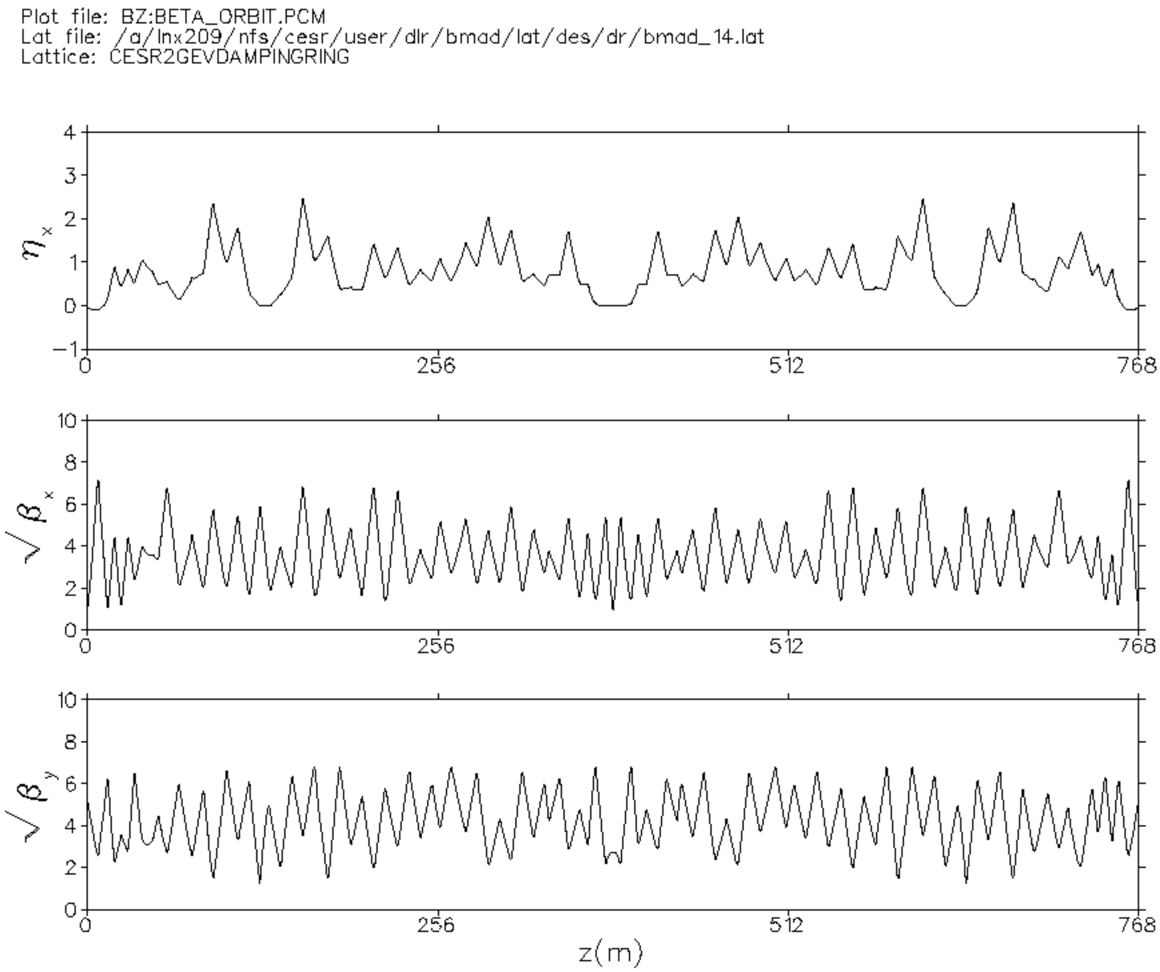

Figure 1: CesrTF Lattice Parameters

The corresponding values from a detailed *BMAD* [7] simulation are

$$\begin{aligned} \mathcal{I}_2 &= 0.97 \text{ m}^{-1} \\ \mathcal{I}_5 &= 1.46 \times 10^{-4} \text{ m}^{-1} \\ \varepsilon_x &= 0.89 \text{ nm} \end{aligned} \tag{12}$$

which show very good agreement with our analytical estimate.

References

[1] D.L. Rubin, CESR Test Facility 12 wiggler lattice design, available at http://www.lns.cornell.edu/~dlr/damping_ring/cesr2gevdr_12wig.html.

[2] A.W. Chao and M. Tigner (*ed.*), "Handbook of Accelerator Physics and Engineering", Section 3.1.4.4, p. 187, World Scientific, 1998.

[3] Ibid., Section 3.1.4.1, pp. 185–186. World Scientific.

[4] H. Wiedemann, "Particle Acclerator Physics I", 2nd ed., Sect. 10.4, pp. 358–363, 1998.

[5] The result obtained here disagrees by a factor of 4 with that given by Wiedemann in references [2, 4]. Note, however, that the derivation of Minty and Zimmerman [6] agrees with the value given here.

[6] M.G. Minty and F. Zimmerman, "Measurement and Control of Charged Particle Beams", Sect. 4.3.3, pp.122–124, Springer, 2003.

[7] D.C. Sagan, "BMAD Subroutine Library for Charged-Particle Simulations", available at http://www.lns.cornell.edu/~dcs/bmad.