# Timing issues for the ILC damping rings* 

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## Abstract

A recent redesign of the ILC has led to changes in the overall configuration as well as revised damping ring parameters. In addition, further understanding of the factors affecting the performance of the damping ring has led to a reconsideration of the constraints on the damping ring. This report summarizes the new constraints, examines the implications of these constraints and the new configuration, and describes possible operating modes of the damping ring. The most promising choices of ring circumference and thus harmonic number are enumerated and examined in detail.

## INTRODUCTION

As part of the Reference Design Report for the ILC [1], the configuration of the machine has been altered to achieve significant cost savings without adversely affecting its capabilities. The revised configuration is shown in Fig. 1 [2]. The most prominent change is that there is a single IR which is now enclosed by the two damping rings. The two damping rings occupy the same tunnel, with the beams in each ring circulating in opposite directions. Overall, the rings will be near mirror images of each other. For each ring, the extraction line is roughly opposite the injection line.
Besides significant differences in the layout of the ILC, several key parameters have also changed. The beam energy in the damping ring is 5 GeV , the 650 MHz RF operates at 24 MV , for a natural bunch length of 9 mm (up from 6 mm ), and the momentum compaction factor is $4.2 \times 10^{-4}$. The most current design for the damping ring lattice has 6 arc sections, 4 short straights with wigglers, and two long straights for injection and extraction. The circumference is 6-7 km but the exact harmonic number, $h$, has not been finalized. Part of this report is a list of suggested values for $h$, along with the choices for operating modes available for each value of $h$.

## CONSTRAINTS

The main constraints required for proper timing of bunches are given in Ref. [3], and are summarized below. A newly considered constraint not included in this document is imposed by the subharmonic buncher of the electron source. This works best when the ratio of bunch spacing to the linac RF period is divisible by powers of 2 and 3 . One factor of two arises automatically because the linac RF period is half of the damping ring RF period. The nominal

[^0]constraint is that this ratio must be divisible by 24 , although additional factors of 3 would be desirable. This report will focus on designs where the bunches are injected (as well as extracted) from the damping ring with a uniform period $k_{b}$, where $k_{b}$ is the ratio between the bunch spacing in the linac and the damping ring RF period. Because the RF period in the damping ring is twice that of the RF in the linac, we require that $k_{b}$ be divisible by 12 based on the requirements of the subharmonic buncher. Furthermore, in order to avoid injecting two bunches in overlapping positions, it is necessary for the lowest common multiple of $h$ and $k_{b}$ to be $>k_{b} N_{b}$, where $N_{b}$ is the total number of bunches. This constraint is fulfilled for all parameters we are considering, however.

Within the damping ring, the bunches should form a sequence of $p$ bunch trains. These bunch trains will be laid out over multiple turns of the damping ring. Each train should consist of no more than 50 bunches, although in practice this number might depend on the charge per bunch. The spacing between bunch trains should be at least 40 ns , or $26 \tau_{\text {RF }}$. Thus, there should be at least 25 empty buckets separating the tail of one bunch train and the head of the next bunch train. We define the minimum gap spacing $g_{0}=26$. Within a bunch train, each bunch must be surrounded by at least one empty bucket on either side, in other words bunches must be separated by at least 3.08 ns . The separation of bunches within a train will be denoted $n_{b}$, so we require $n_{b} \geq 2$. This constraint is related to the rise and fall time of the kicker magnets, to ensure that a single bunch can be extracted without perturbing adjacent bunches. Larger bunch separation would be desirable in order to simplify the design of the kicker magnets. The maximum kicker repetition rate is roughly 6 MHz , which implies that $k_{b} \geq 108$.

Other constraints are imposed by the desire to maximize the ideal luminosity while at the same time limiting instabilities which may degrade this luminosity. The average current in the linac pulse is chosen to be exactly 9 mA . The nominal pulse duration is $980 \mu \mathrm{~s}$, for a total of $5.5 \times 10^{13}$ particles per pulse. The maximum number of particles per bunch is taken to be $2.3 \times 10^{10}$.

To generate a fill pattern of $p$ bunch trains, with spacing $n_{b}$ within each bunch train, it is necessary for $p k_{b}=$ $q h-n_{b}$, where p and q are relatively prime. Here, $q$ is the number of revolutions through the ring before the first bunch kicked in gets a "partner" kicked in $n_{b}$ periods in front of it, extending the bunch train. First, the tail of each bunch train is positioned, then leading bunches are added to each bunch train in turn. The reason for the minus sign above is to have bunch trains extracted starting from the tail of the train and proceeding to the head. This allows the


Figure 1: Revised schematic of the ILC, taken from the ILCDR07 workshop. Figure courtesy of A. Wolski.
requirement on the fall time of the kicker magnets to be relaxed, as short rise times are easier to achieve. Note that if bunches are added until the gaps between bunch trains are completely filled, the bunch spacing may be $<n_{b}$. However, the primary mode of operation we consider is to keep the number of bunches sufficiently low that the gaps are all $\geq g_{0}$. The spacing between the tail of one train and the head of the next is not necessarily a multiple of $n_{b}$, which is what allows us to have $n_{b}>i$, where $i$ is the greatest common factor of $k_{b}$ and $h$.

In order to have the required minimum gap size between each bunch train, the total number of buckets contained within the ring, $h$, must satisfy

$$
\begin{equation*}
h \geq N_{b} n_{b}+p\left(g_{0}-n_{b}\right) \tag{1}
\end{equation*}
$$

Expressing $N_{b}=1+\tau f / k_{b}$, where $\tau$ is the pulse duration in the linac, $f$ is the RF frequency in the damping ring, and $k_{b}$ is the kicker spacing relative to the damping ring RF, and using $p k_{b}=q h-n_{b}$, we can find the following constraint on the value of $k_{b}$ :

$$
\begin{equation*}
k_{b} \geq \frac{n_{b} \tau f}{h}+q\left(g_{0}-n_{b}\right) \tag{2}
\end{equation*}
$$

This, together with the limit on charge per bunch, restricts the allowed values of $k_{b}$ to a narrow range. In fact, for $n_{b}=$ 6 , even for $q=1$ the only possible values are $k_{b} \geq 276$. For a given value of $n_{b}$, as $q$ is increased the minimum $k_{b}$ increases as well.

The most typical filling pattern uses $q=1$, so that each bunch train has one bunch added to it with each rotation period of the ring. Because there can be at most 50 bunches per train, this means that the pulse in the linac can only have a duration of 50 rotation periods. If $h<12740$, then this means that $\tau<980 \mu \mathrm{~s}$. Shorter rings would require either a shorter pulse duration in the linac or allowing more than 50 bunches per bunch train inside the damping ring. For the few examples below which have smaller values of $h$, shorter pulse durations will be assumed.

Additionally, for $k_{b}=108$, even for $n_{b}=2$ the maximum number of bunches per train allowed under the typical $q=1$ mode is $1+\left(k_{b}-g_{0}\right) / n_{b}=42$. This won't allow the full $980 \mu$ s pulse. Even for $k_{b}=120$, the maximum number of bunches per train is 48 , which will require shortened bunch trains if $h<13270$. As will be shown
below, $k_{b}=108$ is only an option for the high end of the choices for $h$. parameters anyway.

For the nominal beam current, the maximum particles per bunch of $2.3 \times 10^{10}$, together with the constraint that $k_{b}$ be divisible by 12 , requires $k_{b} \leq 264$. Using a more aggressive limit of $2.5 \times 10^{10}$ particles per bunch would allow values of $k_{b}$ up to 288 .

## MAIN OPERATING MODE

We have basically restricted our attention to $k_{b}=12 a$, where $a$ is an integer between 9 and 22. Thus, we are looking for choices of $p k_{b} / 12$ which contain a maximum number of divisors in this range. The most obvious choices are multiples of 60 , and for $q=1$ they should lie somewhere between 900 and 1300. The best choice for having many factors is 1260 , which leads to $h=15120+n_{b}$, but other options are also potentially interesting. Some promising choices which do not fit this pattern are $h-n_{b}=14784$, 13860, 13440, and 12096. Below, we focus on the best options with the most flexibility under the constraints.

There are several different types of flexibility that are desirable for the damping ring operation. First, in the linac it is useful to have different choices for the bunch spacing, in other words different choices of $k_{b}$. Within the damping ring, it is useful to have different numbers of bunch trains, $p$; this comes naturally from different choices of $k_{b}$ through $p k_{b}=q h-n_{b}$. It is also worthwhile to have different options for the value of $n_{b}$, as the sensitivity of collective effects to $n_{b}$ is not well known. Unfortunately, increasing $n_{b}$ tends to reduce the number of acceptable values for $k_{b}$. Furthermore, those values of $h$ which allow for multiple choices of $n_{b}$ tend to only have 1 or 2 choices of $k_{b}$ for each each value of $n_{b}$. There are a few cases with $n_{b}=2$ that also work for $n_{b}=4$ simply by doubling $k_{b}$. There are almost no examples which allow for switching between $n_{b}=2$ and $n_{b}=3$, or between $n_{b}=3$ and $n_{b}=4$, and the only one which doesn't just offer one value of each or require $k_{b}>264$ is $h=12013$. Flexibility in $n_{b}$ can also be achieved by varying $h$ by 1 or 2 , but this is probably too large a shift to be achieved by the tunable delay sections as they are currently envisioned.

Table 1: Options for $h$ parameter and different operating modes.

| $h$ | $n_{b}$ | $k_{b}$ | $p$ | $q$ | $i$ | $N_{b}$ | $\tau(\mu \mathrm{s})$ | $N_{p}\left(10^{10}\right)$ | $M_{\text {max }}$ | $g_{\text {min }}$ | $L$ (m) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Single value for $n_{b}$ : |  |  |  |  |  |  |  |  |  |  |  |
| 15122 | 2 | 252 | 60 | 1 | 2 | 2528 | 980 | 2.18 | 43 | 169 | 6975 |
| 15122 | 2 | 240 | 63 | 1 | 2 | 2654 | 980 | 2.07 | 43 | 157 | 6975 |
| 15122 | 2 | 216 | 70 | 1 | 2 | 2949 | 980 | 1.87 | 43 | 133 | 6975 |
| 15122 | 2 | 180 | 84 | 1 | 2 | 3539 | 980 | 1.56 | 43 | 97 | 6975 |
| 15122 | 2 | 168 | 90 | 1 | 2 | 3792 | 980 | 1.45 | 43 | 85 | 6975 |
| 15122 | 2 | 144 | 105 | 1 | 2 | 4424 | 980 | 1.24 | 43 | 61 | 6975 |
| 15122 | 2 | 120 | 126 | 1 | 2 | 5308 | 980 | 1.04 | 43 | 37 | 6975 |
| 15122 | 2 | 108 | 140 | 1 | 2 | 5881 | 977 | 0.93 | 43 | 26 | 6975 |
| 15123 | 3 | 252 | 60 | 1 | 3 | 2528 | 980 | 2.18 | 43 | 128 | 6975 |
| 15123 | 3 | 240 | 63 | 1 | 3 | 2654 | 980 | 2.07 | 43 | 116 | 6975 |
| 15123 | 3 | 216 | 70 | 1 | 3 | 2949 | 980 | 1.87 | 43 | 92 | 6975 |
| 15123 | 3 | 180 | 84 | 1 | 3 | 3539 | 980 | 1.56 | 43 | 56 | 6975 |
| 15123 | 3 | 168 | 90 | 1 | 3 | 3792 | 980 | 1.45 | 43 | 44 | 6975 |
| 14402 | 2 | 240 | 60 | 1 | 2 | 2654 | 980 | 2.07 | 45 | 153 | 6642 |
| 14402 | 2 | 192 | 75 | 1 | 2 | 3318 | 980 | 1.66 | 45 | 105 | 6642 |
| 14402 | 2 | 180 | 80 | 1 | 2 | 3539 | 980 | 1.56 | 45 | 93 | 6642 |
| 14402 | 2 | 144 | 100 | 1 | 2 | 4424 | 980 | 1.24 | 45 | 57 | 6642 |
| 14402 | 2 | 120 | 120 | 1 | 2 | 5308 | 980 | 1.04 | 45 | 33 | 6642 |
| 12962 | 2 | 240 | 54 | 1 | 2 | 2654 | 980 | 2.07 | 50 | 143 | 5978 |
| 12962 | 2 | 216 | 60 | 1 | 2 | 2949 | 980 | 1.87 | 50 | 119 | 5978 |
| 12962 | 2 | 180 | 72 | 1 | 2 | 3539 | 980 | 1.56 | 50 | 83 | 5978 |
| 12962 | 2 | 144 | 90 | 1 | 2 | 4424 | 980 | 1.24 | 50 | 47 | 5978 |
| 12962 | 2 | 120 | 108 | 1 | 2 | 5185 | 957 | 1.04 | 49 | 26 | 5978 |
| Two choices for $n_{b}$ : |  |  |  |  |  |  |  |  |  |  |  |
| 14042 | 2 | 216 | 65 | 1 | 2 | 2949 | 980 | 1.87 | 46 | 127 | 6476 |
| 14042 | 2 | 180 | 78 | 1 | 2 | 3539 | 980 | 1.56 | 46 | 91 | 6476 |
| 14042 | 2 | 156 | 90 | 1 | 2 | 4083 | 980 | 1.35 | 46 | 67 | 6476 |
| 14042 | 2 | 120 | 117 | 1 | 2 | 5308 | 980 | 1.04 | 46 | 31 | 6476 |
| 14042 | 4 | 240 | 117 | 2 | 2 | 2654 | 980 | 2.07 | 23 | 33 | 6476 |
| 13862 | 2 | 252 | 55 | 1 | 2 | 2528 | 980 | 2.18 | 46 | 162 | 6393 |
| 13862 | 2 | 180 | 77 | 1 | 2 | 3539 | 980 | 1.56 | 46 | 90 | 6393 |
| 13862 | 2 | 132 | 105 | 1 | 2 | 4826 | 980 | 1.14 | 46 | 42 | 6393 |
| 13862 | 4 | 264 | 105 | 2 | 2 | 2413 | 980 | 2.28 | 23 | 44 | 6393 |
| 12602 | 2 | 252 | 50 | 1 | 2 | 2500 | 969 | 2.18 | 50 | 154 | 5812 |
| 12602 | 2 | 180 | 70 | 1 | 2 | 3500 | 969 | 1.56 | 50 | 82 | 5812 |
| 12602 | 2 | 168 | 75 | 1 | 2 | 3750 | 969 | 1.45 | 50 | 70 | 5812 |
| 12602 | 4 | 240 | 105 | 2 | 2 | 2573 | 950 | 2.07 | 25 | 26 | 5812 |
| 15125 | 3 | 228 | 199 | 3 | 1 | 2794 | 980 | 1.97 | 15 | 36 | 6976 |
| 15125 | 5 | 252 | 60 | 1 | 1 | 2528 | 980 | 2.18 | 43 | 46 | 6976 |
| 15125 | 5 | 240 | 63 | 1 | 5 | 2654 | 980 | 2.07 | 43 | 34 | 6976 |
| 12013 | 2 | 264 | 91 | 2 | 1 | 2413 | 980 | 2.28 | 27 | 80 | 5541 |
| 12013 | 2 | 168 | 143 | 2 | 1 | 3792 | 980 | 1.45 | 27 | 32 | 5541 |
| 12013 | 3 | 252 | 143 | 3 | 1 | 2528 | 980 | 2.18 | 18 | 33 | 5541 |

The best choices for this main operating mode, where the pulse in the linac is completely regular with no gaps, are given in Table 1. If it is required to have $n_{b}>2, h=12099$ or $h=15123$ are the best options. The table also gives the greatest common factor, $i$, of $k_{b}$ and $h$. Larger values of $i$ may simplify feedback systems, and are also desirable, but in most cases $i=n_{b}$. In a few examples, $i=n_{b} / 2$ or $i=1$. In any case, unless the kicker is to have a variable period, it is necessary for $n_{b}$ to be a multiple of $i$, so large values of $i$ require large values of $n_{b}$.

The biggest "culprits" for the lack of options are the requirement that $k_{b}$ be divisible by 12 , the limit of maximum charge per bunch of 3.7 nC , which forces $k_{b} \leq 264$, and the pulse duration of $980 \mu \mathrm{~s}$. The pulse duration, together with the requirement that there be at most 50 bunches per train in the damping ring, eliminates most options when the ring circumference is less than 5875 m , or $h \leq 12740$. The example where $h=12602$ only worked by reducing the pulse duration to $969 \mu \mathrm{~s}$, although smaller rings are undesirable for other reasons. The narrow range of choices for $k_{b}$ and $h$ eliminate the possibility of independent tuning of both $n_{b}$ and $k_{b}$ for fixed circumference. Because of uncertainty in how instabilities are related to the bunch spacing, $h=14042$ seems to be the best option. However, a ring with $h=15122$ has many options for $n_{b}=2$, and it may be possible to modify the ring so that $h=15123$, which has various options for $n_{b}=3$. Switching back and forth between these two circumferences would probably not be possible, however. The last two options, $h=15125$ and $h=12013$, have been included because they allow for combinations of $n_{b}$ which cannot be achieved in any other way, which might be useful if new difficulties become apparent either with $n_{b}=2$ or $n_{b}=4$.

## ADDITIONAL OPTIONS THROUGH RELAXED CONSTRAINTS

Changing or eliminating constraints on the damping ring parameters or operating mode will clearly allow for many new options. Allowing for more charge per bunch or reducing the average current will increase the maximum value of $k_{b}$, while reducing the pulse duration or the minimum gap between bunch trains will allow for smaller values of $k_{b}$. Also, instead of extracting bunches from the tail to the head of each train, they can be extracted in the reverse order by choosing $p k_{b}=q h+n_{b}$, or in a more complicated order by using large values of $q$ so that adjacent bunches are extracted at very different times. While many new options can thus be found, most of these choices will have the same values of $n_{b}$ and $i$. In fact, restricting $k_{b}$ to be a multiple of 12 pushes one to values of $h$ which do not have many small prime factors, and thus greatly restricts the possible values of $n_{b}$ and $i$.

It is useful to consider what possibilities exist for a given value of $h \bmod 12$, given that $k_{b}$ must be a multiple of 12 . If $h \bmod 12=0$ or 11 , then $n_{b}=11$ or 12 or higher, which is not going to work. Otherwise, $n_{b}=h \bmod 12$ is
an option, usually with $i=n_{b}$, but again $n_{b}>6$ is not going to work under the above constraints. There are only a few alternative situations which satisfy all of the constraints, in particular Eq. (2), which are listed below. When $h \bmod 12=1, q=1$ will yield bunches which are too close together and instead we require $n_{b}=q=2,3$, or 4 ; in all of these cases, $i=1$. When $h \bmod 12=2$, then $i=2$ and there are two possibilities besides $q=1, n_{b}=2$ : $q=7, n_{b}=2$, or $q=2, n_{b}=4$. When $h \bmod 12=3$, then $n_{b}=i=3$ and $q=5$ is a possibility. When $h$ $\bmod 12=4$, then $n_{b}=i=4$ and $q=4$ is a possibility. When $h \bmod 12=5$, then when $n_{b}=5$, and $q=1, i$ can be either 1 or 5 depending on $k_{b}$; otherwise, $n_{b}=q=3$ and $i=1$. When $h \bmod 12=7$, then $n_{b}=q=2$ or 4 , and $i=1$. When $h \bmod 12=8$, then $n_{b}=i=4$ and $q=2$. When $h \bmod 12=9$, then $n_{b}=i=3$ and $q=3$. When $h \bmod 12=10$, then $i=2$ and either $n_{b}=q=4$ or $n_{b}=2$ and $q=5$. These possibilities plus the choices for $q=1$ are detailed in Table 2. The small number of options for each case explains the difficulty in finding different choices for $n_{b}$ and $i$ for a given value of $h$. Most of the alternative options beyond $n_{b}=h \bmod 12$ are only possible for the largest allowed values of $k_{b}$, yielding at most a single alternative. Furthermore, if we want to be able to choose between $i=2,3$, and 4 , then that implies $h \bmod 12=0$, which as stated above is a terrible choice for finding solutions where $k_{b}$ is a multiple of 12 .

Table 2: Parameter choices for different values of $h$ $\bmod 12$, considering only $2 \leq n_{b} \leq 6$ and small values of $q$.

| $h \bmod 12$ | $n_{b}$ | $q$ | $i$ |
| :---: | :---: | :---: | :---: |
| 0 | - | - | - |
| 1 | 2 | 2 | 1 |
|  | 3 | 3 | 1 |
|  | 4 | 4 | 1 |
| 2 | 2 | 1 | 2 |
|  | 2 | 7 | 2 |
|  | 4 | 2 | 2 |
| 3 | 3 | 1 | 3 |
|  | 3 | 5 | 3 |
| 4 | 4 | 1 | 4 |
|  | 4 | 4 | 4 |
| 5 | 5 | 1 | 1 or 5 |
|  | 3 | 3 | 1 |
| 6 | 6 | 1 | 6 |
| 7 | 2 | 2 | 1 |
|  | 4 | 4 | 1 |
| 8 | 4 | 2 | 4 |
| 9 | 3 | 3 | 3 |
| 10 | 2 | 5 | 2 |
|  | 4 | 4 | 2 |
| 11 | - | - | - |

Instead, we consider on other modes of operation. Most of the examples given above with $n_{b}=4$ were obtained by taking an $n_{b}=2, q=1$ solution, and doubling $k_{b}$ while keeping $p$ fixed to yield an $n_{b}=4, q=2$ solution. However, this is only possible if $p$ is odd, otherwise doubling $k_{b}$ yields the solution $(p / 2)\left(2 k_{b}\right)=q h-n_{b}$, and $n_{b}$ is unchanged. For these situations, allowing gaps in the linac pulse allows for new solutions with the bunch spacing in the damping ring doubled. If the linac pulse is alternately run as normal or switched off every rotation period of the damping ring (roughly $20 \mu \mathrm{~s}$ ), then within each bunch train, every other bunch is simply left out. The bunch spacing is thus doubled, while $p$ and $k_{b}$ are left the same. As a result, however, the linac pulse is split up into $20 \mu$ s pieces, which alternate between having 18 mA (twice the nominal current) and no current at all. The charge per bunch must also be doubled to maintain the same total charge per pulse, as half of the bunches are left out. The larger current may present problems with instabilities, but this scheme has the advantage of a reasonably simple timing structure for the electron source. This method works when the original particles per bunch is less than half the maximum and $p$ is even (if $p$ is odd then this method is not necessary). For example, with $h=15122$ and $k_{b}=120$ or $k_{b}=108$, we can create bunch trains with $n_{b}=4$ in this way. Similarly, with either $h=14402$ or 12962 , one option having $n_{b}=4$ can be accessed in this way. This technique also works well for $h=12013$, because "solutions" with $q=1$ and $n_{b}=1$ can be converted to allowing for $n_{b}=2$. In particular, $k_{b}=132$ or $k_{b}=156$ are possible modes of operation, although $k_{b}=156$ will require too much charge per bunch for the average current to be maintained at 9 mA .

Further variations can be achieved by relaxing the condition on the sequence in which bunches are extracted. Bunch trains can extend until they come close to overlapping other bunch trains, and then either the gaps can be widened by eliminating key bunches, or an entirely new set of gaps can be created. By eliminating bunches to form fewer than $p$ gaps, longer bunch trains can be formed; this is especially useful for small $k_{b}$. This method can be used even if there are a huge number of very small overlapping bunch trains, although the ordering in which bunches are extracted will be very irregular. Even when a choice of $k_{b}$ leads to extremely large values of $p$ and $q$, the bunch trains can be redefined in this way, especially in cases where $i>1$ because then all bunches will at least be separated by the required 3.08 ns . The resulting $n_{b}$ will typically be equal to $i$, although it might be possible to have $n_{b}=2 i$ in special cases.

For most of the cases considered in the table, the main value for $n_{b}$ is equal to $i$ and is the only small prime factor of $h$. The most extreme case is 15122 , half of which is a prime number itself. Thus, even the most convoluted timing scheme will result in $n_{b}=2$ or $n_{b}=4$, and the only choices for $i$ are 1 or 2 . Thus, although this choice for $h$ works very well under the constraints, relaxing those constraints does not yield substantially new solutions. Even if
we allow for odd values of $k_{b}$, there is no reasonable way to obtain $n_{b}=3$ or $n_{b}=5$, because $q h-3$ and $q h-5$ have no factors in the required range of values for $k_{b}$, for any reasonable values of $q$. On the other hand, for $h=14402$, we can have $n_{b}=3, q=1$, and $k_{b}=187$, which works within all other constraints, yielding $1.62 \times 10^{10}$ particles per bunch. Here, $k_{b}$ is not divisible by either 2 or 3 ; except for the case where $h \bmod 12=5, k_{b}$ will never be divisible by 2 or 3 in these extended examples. The value for $i$ will always be 1 . For $h=12962$, we have $161 * 161=2 h-3$, so this allows for $n_{b}=3$. Unfortunately, this value for $k_{b}$ is too small and the maximum pulse duration will be $764 \mu \mathrm{~s}$. For $h=14042$, we can have $n_{b}=3, q=1$, and $k_{b}=139$, which works but with a pulse duration of $835 \mu \mathrm{~s}$.

There are several extended options for $h=15125: n_{b}=$ $2, k_{b}=213$, yields $1.62 \times 10^{10}$ particles per bunch, $n_{b}=2$, $k_{b}=152$, yields $1.31 \times 10^{10}$ particles per bunch. These are nice examples because $k_{b}$ at least has one factor of 2 or 3. Of course, there are also additional options for $n_{b}=3$ and 5.

It is possible that some options which were neglected, would work very well after relaxing one or more of the constraints. However, the focus here is on finding values of $h$ which work best under the given constraints, and indicate how those options might look if some constraints were removed.

## POSITRON GENERATION BY ELECTRONS

There seems to be very little room to maneuver the electron bunches so each one extracted generates a positron bunch that drops into the ring right where the positron bunch it collided with came from. Other than very carefully choosing the circumference, the only real options seem to be adding delay lines that could be longer than 1 km , or changing the arc length between the injection and extraction points. If, instead of being on exactly opposite ends of the linac, the injection and extraction lines are each shifted by $\Delta_{1}$ damping ring RF periods, then it will take $h / 2-2 \Delta_{1}$ RF periods for a bunch to travel between the injection and extraction points. In order for the timing scheme to work, we must have $L=h / 2+2 \Delta_{1}+N_{c} h$, where $N_{c}$ is any integer and $L$ is the time between electron extraction and injection of the positrons generated by those same electrons (in units of length, that's roughly 29 km ). You could also have $\Delta_{2}$ separating the electron injection and positron extraction (and vice versa), but all that does is replace $2 \Delta_{1}$ with $2 \Delta_{1}+\Delta_{2}$.

## REFERENCES

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